Chapter 40
Bending of beams

Beam theory

The bending of ships can be likened to the bending of beams in many cases. This chapter shows the procedures employed with beam theory.

The problem of calculating the necessary strength of ships is made difficult by the many and varied forces to which the ship structure is subjected during its lifetime. These forces may be divided into two groups, namely statical forces and dynamical forces.

The statical forces are due to:

1 The weight of the structure which varies throughout the length of the ship.
2 Buoyancy forces, which vary over each unit length of the ship and are constantly varying in a seaway.
3 Direct hydrostatic pressure.
4 Concentrated local weights such as machinery, masts, derricks, winches, etc.

The dynamical forces are due to:

1 Pitching, heaving and rolling,
2 Wind and waves.

These forces cause bending in several planes and local strains are set up due to concentrated loads. The effects are aggravated by structural discontinuities.

The purpose of the present chapter is to consider the cause of longitudinal bending and its effect upon structures.

Stresses

A stress is the mutual actual between the parts of a material to preserve their relative positions when external loads are applied to the material.

Thus, whenever external loads are applied to a material stresses are created within the material.

Tensile and compressive stresses

When an external load is applied to a material in such a way as to cause an extension of the material it is called a 'tensile' load, whilst an external load tending to cause compression of the material is a 'compressive' load.

Figure 40.1 shows a piece of solid material of cylindrical section to which an external load W is applied. In the first case the load tends to cause an extension of the material and is therefore a tensile load. The applied load creates stresses within the material and these stresses are called 'tensile' stresses. In the second case the load applied is one of compression and the consequent stresses within the material are called 'compressive' stresses.

When a tensile or compressive external load is applied to a material the material will remain in equilibrium only so long as the internal forces can resist the stresses created.

Shearing stresses

A shearing stress is a stress within a material which tends to break or shear the material across.

Figure 40.2(a) and 40.2(b) illustrate shearing stresses which act normally to the axis of the material.

In the following text when the direction of a shearing stress is such that

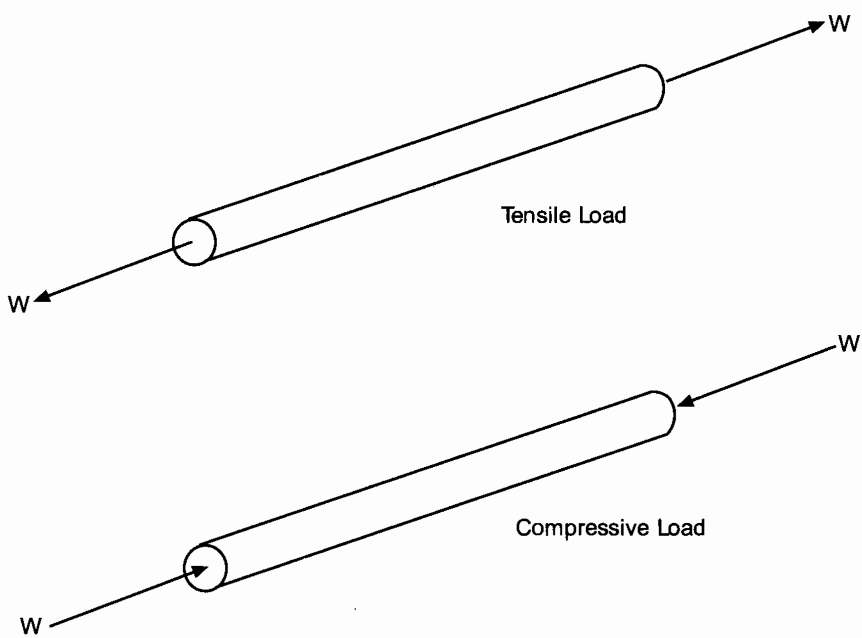


Fig. 40.1

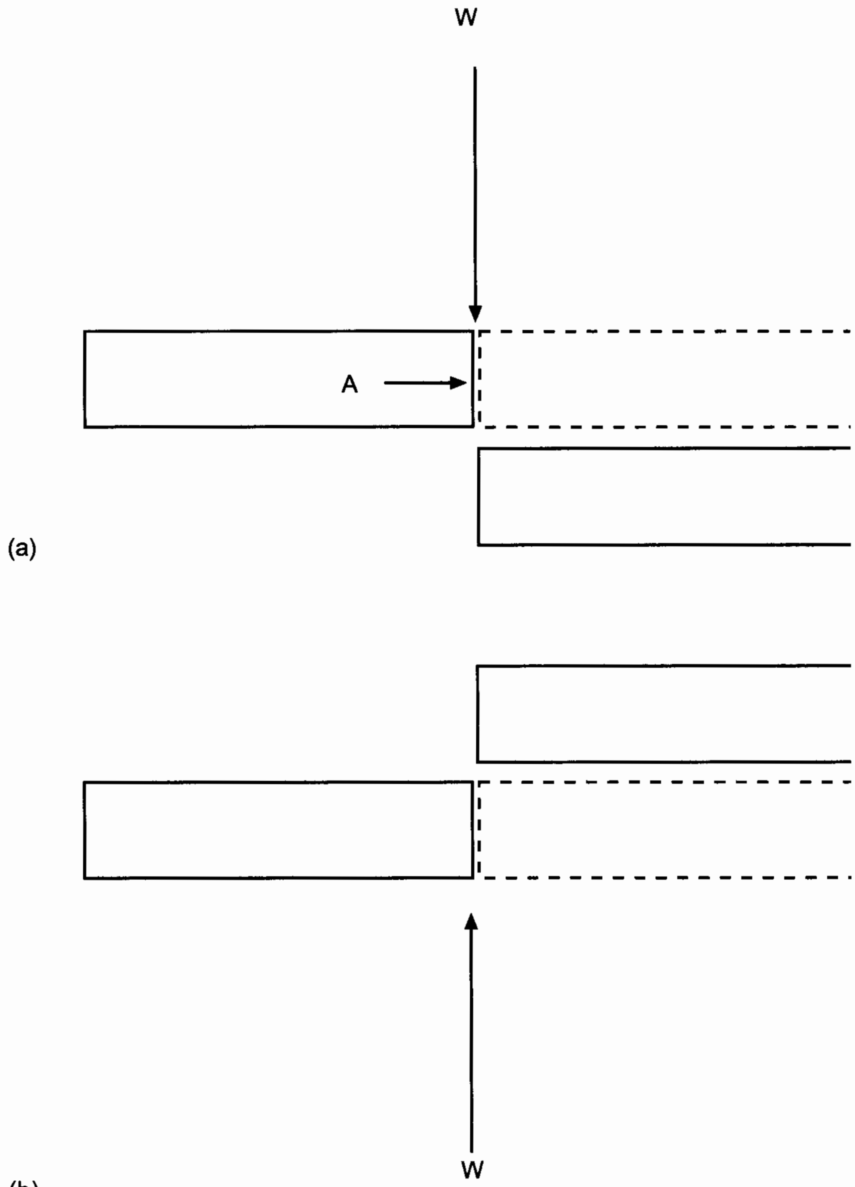


Fig. 40.2

the section on the right-hand side of the material tends to move downwards, as shown in Figure 40.2(a), the stress is considered to be positive, and when the direction of a stress is such that the section on the right-hand side tends to move upwards as shown in Figure 40.2(b), the shearing stress is considered to be negative.

Shearing stresses are resisted by the material but shearing will take place when the shear stress reaches the ultimate shear stress of the material.

Complementary stress

It has already been stated that when a direct load is applied to a material stresses are created within the material and that the material will remain in equilibrium only so long as the internal forces can resist the stresses created.

Let the bar in Figure 40.3(a) be subjected to tensile load W and imagine the bar to be divided into two parts at section AB.

For equilibrium, the force W on the left-hand side of the bar is balanced by an equal force acting to the right at section AB. The balancing force is supplied by the individual molecular forces which represent the action of the molecules on the right-hand side of the section on those of the left-hand section. Similarly, the force W on the right-hand side of the bar is balanced by the individual molecular forces to the left of the section. Therefore, when an external load W is applied to the bar, at any section normal to the axis of the bar, there are equal and opposite internal forces acting, each of which balances the external force W. The magnitude of the internal forces per unit area of cross-section is called the stress. When the section is well removed from the point of application of the external load then the stress may be considered constant at all parts of the section and may be found by the formula:

$$\text{Stress (f)} = \frac{\text{Load (W)}}{\text{Area (A)}} \quad \therefore f = \frac{W}{A}$$

Let us now consider the stresses created by the external load W in a section which is inclined to the axis of the bar. For example, let section CD in Figure 40.3(b) be inclined at an angle θ to the normal to the axis of the bar and let the section be sufficiently removed from the point of application of the load to give uniform stress across the section.

The load transmitted by the section CD, for equilibrium, is equal to the external force W. This load can be resolved into two components, one of which is $W \cdot \cos\theta$ and acts normal to the section, and the other is $W \cdot \sin\theta$ and acts tangential to the section. This shows that for direct tensile or compressive loading of the bar stresses other than normal stresses may be created.

Now let us consider the small block of material abcd in the section on the left-hand side of the plane, as shown in Figure 40.3(c). Let the face ab be coincident with the plane CD and let F_N be the internal force normal to this face and F_T the internal force tangential to the face. For the block to be in equilibrium the left-hand side of the section must provide two stresses on the face cd. These are F_N and F_T. Thus the stress F_N normal to the face ab is balanced by the stress F_N normal to the face cd whilst the two tangential stresses (F_T) on these faces tend to produce clockwise rotation in the block. Rotation can only be prevented if an equal and opposite couple is produced

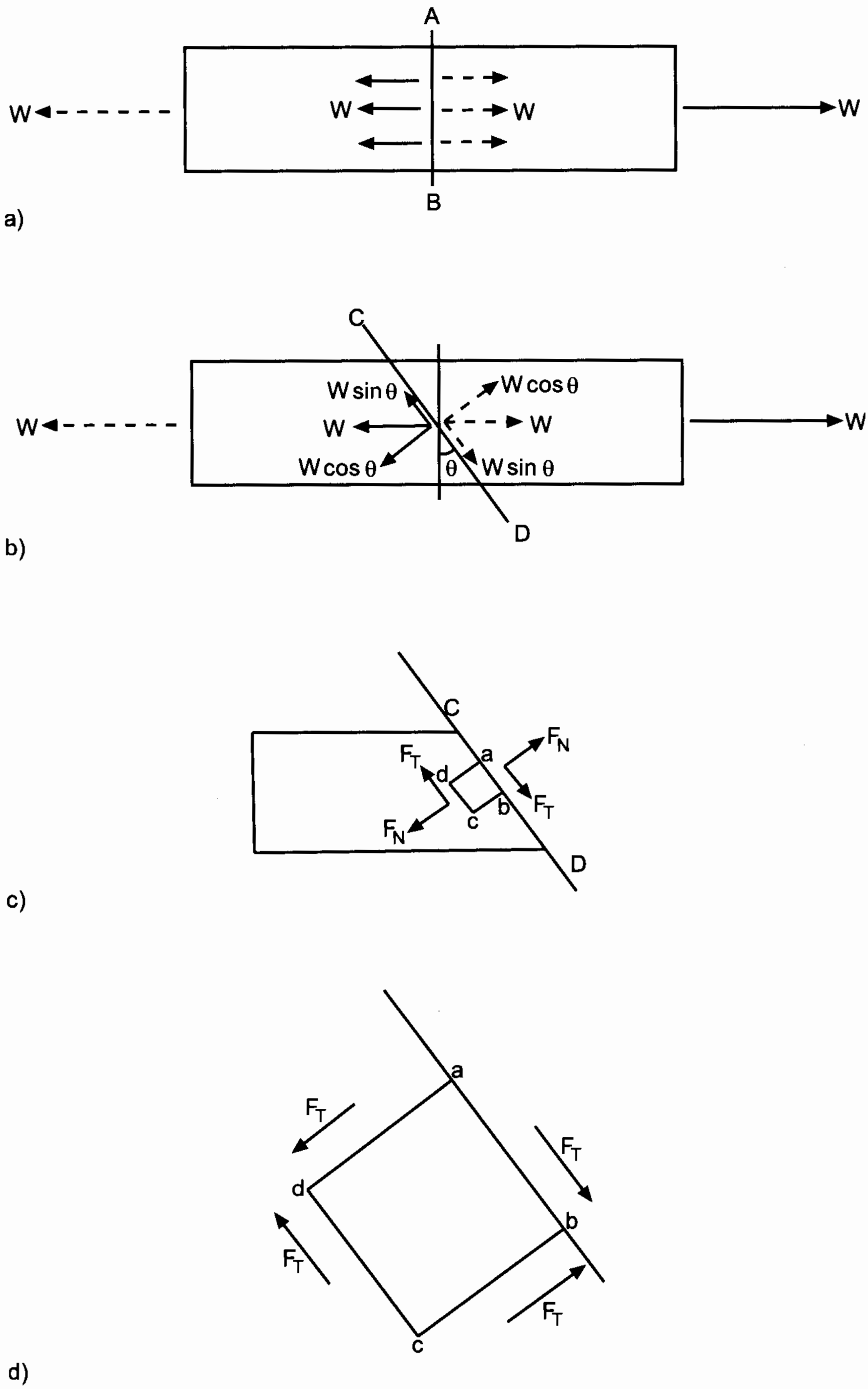

A
B
W
W
W
W
a)
C
D
W sin θ
W cos θ
W
W
W cos θ
θ
W sin θ
W
W
b)
C
D
a
b
c
d
F_N
F_T
F_T
F_N
c)
a
b
c
d
F_T
F_T
F_T
F_T
d)

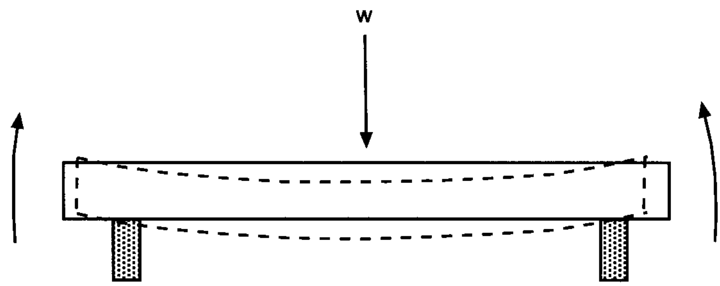
Fig. 40.3

by opposing shearing stresses on the faces ad and bc as shown in Figure 40.3(d).

It can therefore be seen that when shear stresses occur at any plane within a material, equal shear stresses are produced on planes at right angles. These equal and opposing shearing stresses are called 'Complementary' shearing stresses.

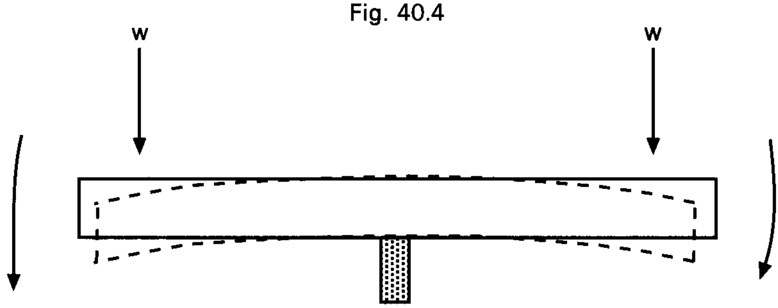
Bending moments in beams

The shear forces and bending moments created within a beam depend upon both the way in which the beam is supported and the way in which it is loaded. The bending moment at any section within the beam is the total moment tending to alter the shape of the beam as shown in Figures 40.4 and 40.5 and is equal to the algebraic sum of the moments of all loads acting between the section concerned and either end of the beam.



Sagging Beam

Fig. 40.4



Hogging Beam

Fig. 40.5

In the following text, when a bending moment tends to cause sagging or downwards bending of the beam as shown in Figure 40.4 it is considered to be a negative bending moment and when it tends to cause hogging or convex upwards bending of the beam, as shown in Figure 40.5, it is considered to be positive. Also, when bending moments are plotted on a graph, positive bending moments are measured below the beam and negative bending moments above.

Shear Force and Bending Moment Diagrams

The shear forces and bending moments created in a beam which is supported and loaded in a particular way can be illustrated graphically.

Consider first the case of cantilevers which are supported at one end only.

Case I

The beam AB in Figure 40.6 is fixed at one end only and carries a weight 'W' at the other end.

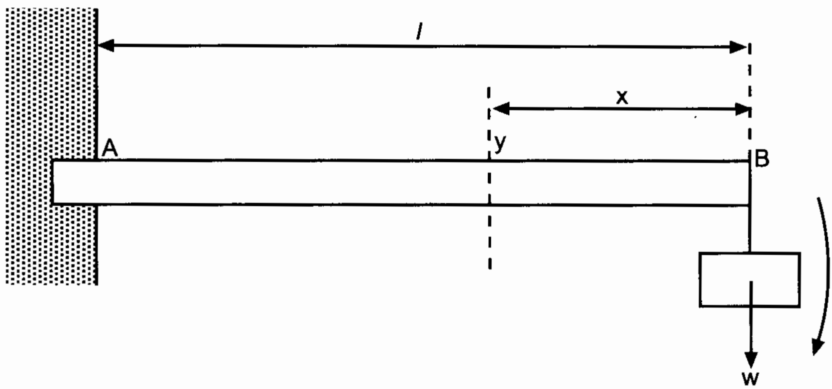


Fig. 40.6

If the weight of the beam is ignored then at any point Y in the beam, which is at distance X from the end B, there is a positive shearing force W and a positive bending moment W × X. There is thus a positive shearing force W at all sections throughout the length of the beam. This is shown graphically in Figure 40.7 where AB represents the length of the beam (*l*), and the ordinate AC, which represents the shearing force at A, is equal to the ordinate BD which represents the shearing force at B.

The bending moment at any section of the beam is the algebraic sum of the moments of forces acting on either side of the section. In the present case, the only force to consider is W which acts downwards through the end B. Thus the bending moment at B is zero and from B towards A the bending moment increases, varying directly as the distance from the end B. The maximum bending moment, which occurs at A, is equal to W × *l*. This is shown graphically in Figure 40.7 by the straight line BGE.

The shearing force and bending moment at any point in the length of the beam can be found from the graph by inspection. For example, at Y the shearing force is represented by the ordinate YF and the bending moment by the ordinate YG.

It should be noted that the bending moment at any point in the beam is equal to the area under the shearing force diagram from the end of the beam to that point. For example, in Figure 40.7, the bending moment at Y is

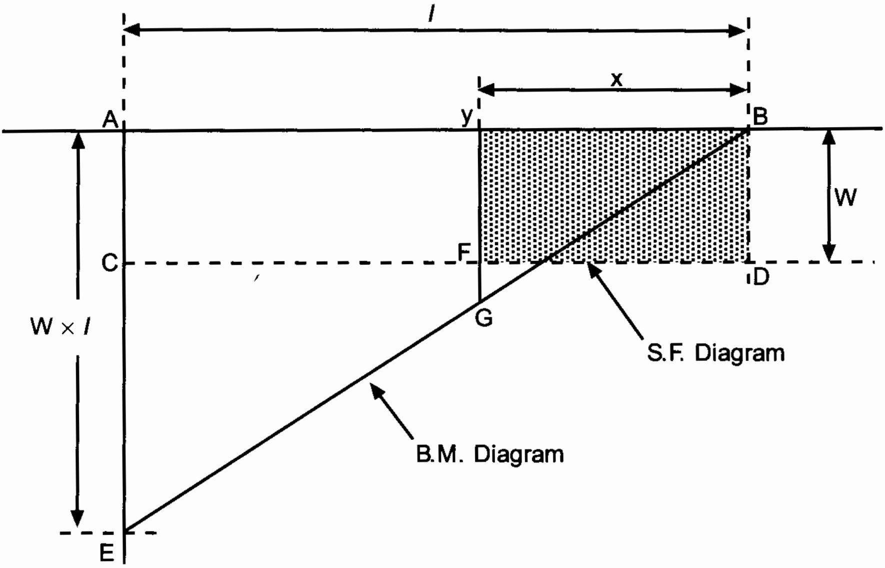


Fig. 40.7

equal to W × X and this, in turn, is equal to the area under the shearing force diagram between the ordinates BD and YF.

Case II

Now consider a solid beam of constant cross-section which is supported at one end as shown in Figure 40.8. Let w be the weight per unit length of the beam.

At any section Y in the beam, which is at distance 'X' from B, there is a positive shearing force wX where wX is the weight of the beam up to that section and, since the weight wX may be taken to act half-way along the length X, there is a bending moment wX × X/2 or $\frac{wX^2}{2}$.

This is shown graphically in Figure 40.9, where AB represents the length of the beam (*l*).

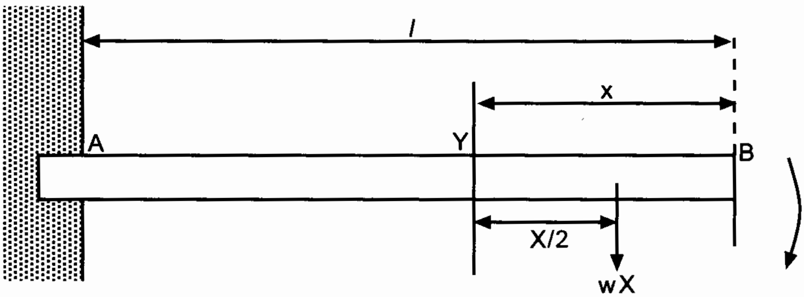


Fig. 40.8

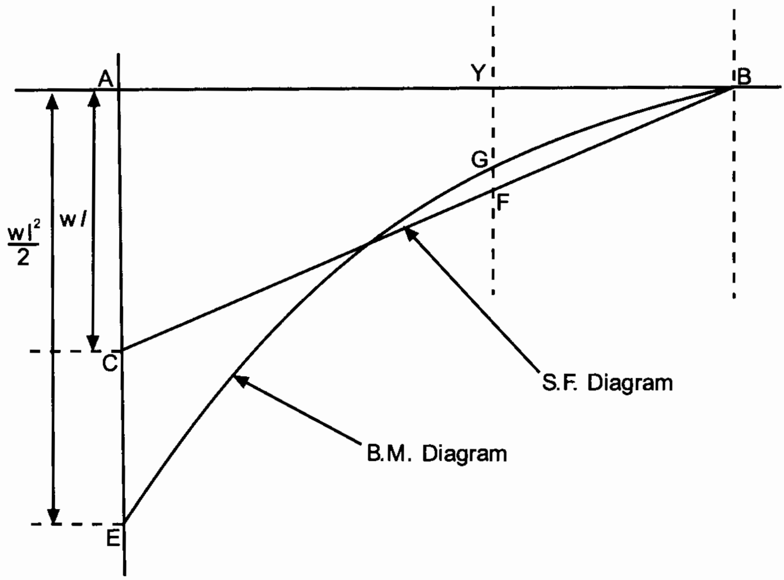


Fig. 40.9

The shearing force at B is zero and then increases towards A, varying directly as X, to reach its maximum value at A of wl. This is represented in Figure 40.9 by the straight line BFC.

The bending moment at any point in the beam is equal to $wX^2/2$. It is therefore zero at B and then increases towards A, varying directly as X^2, to reach its maximum value of $wl^2/2$ at A. The curve of bending moments is therefore a parabola and is shown in Figure 40.9 by the curve BGE.

Since the bending moment at any section is equal to the area under the shearing force diagram from the end of the beam to that section, it follows that the bending moment curve may be drawn by first calculating the area under the shearing force diagram from the end of the beam to various points along it and then plotting these values as ordinates of the curve. For example, at section Y in Figure 40.9 the ordinate YF represents the shearing force at this section (wX), and the area under the shearing force diagram between B and the ordinate FY is equal to $\frac{1}{2} \times wX \times X$ or $wX^2/2$. The ordinate YG could now be drawn to scale to represent this value.

Freely supported beams

Case I

Consider now a beam which is simply supported at its ends, and loaded in the middle as shown in Figure 40.10. In this figure AB represents the length of the beam (l), and W represents the load. If the weight of the beam is neglected then the reaction at each support is equal to W/2, denoted by R_A and R_B.

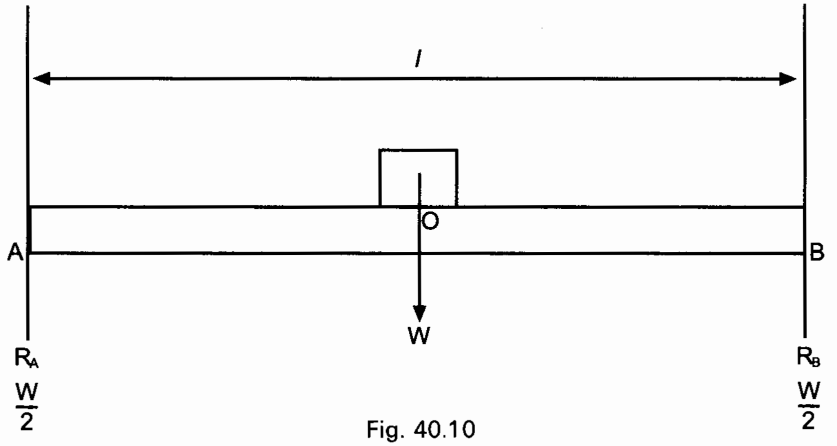


Fig. 40.10

To plot the shearing force diagram first draw two axes of reference as shown in Figure 40.11 with AB representing the length of the beam (l).

Now cover Figure 40.10 with the right hand, fingers pointing to the left, and slowly draw the hand to the right gradually uncovering the figure. At A there is a negative shearing force of W/2 and this is plotted to scale on the graph by the ordinate AC. The shearing force is then constant along the beam to its mid-point O. As the hand is drawn to the right, O is uncovered

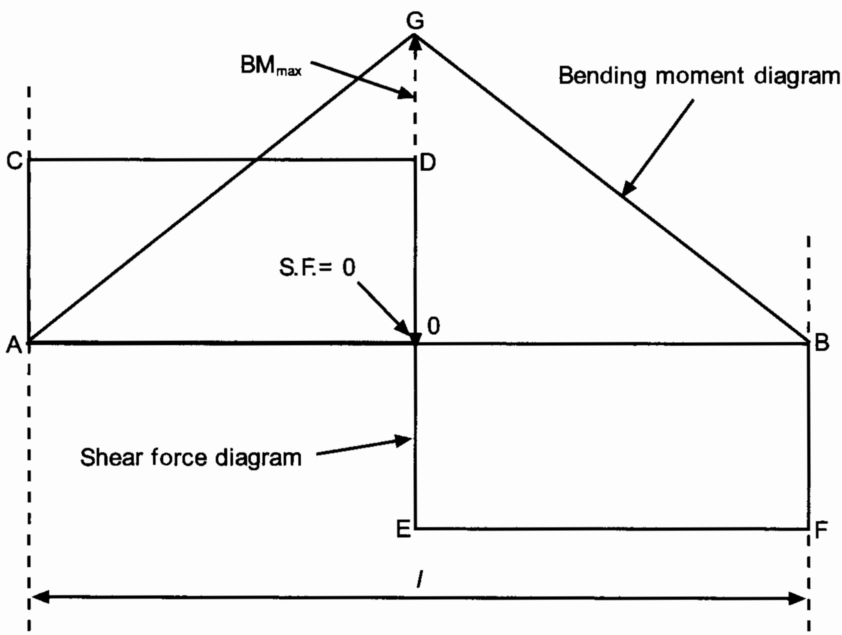


Fig. 40.11

and a force W downwards appears. This must be considered in addition to the force W/2 upwards at A. The resultant is a shearing force of W/2 downwards and this force is then constant from O to B as shown in Figure 40.11 by the straight line EF.

The bending moment diagram can now be drawn in the same way or by first calculating the area under the shearing force diagram between the ordinate AC and various points along the beam and then plotting these values as ordinates on the bending moment diagram. It will be seen that the bending moment is zero at the ends and attains its maximum value at the mid-point in the beam indicated by the ordinate OG. Note BM_{max} occurs when $SF = 0$.

Case II

Now consider a beam of constant cross-sectional area, of length *l*, and weight w per unit length. Let the beam be simply supported at its ends as shown in Figure 40.12, at reactions R_A and R_B.

The total weight of the beam is *wl*. The reaction at each end is equal to $wl/2$, half of the weight of the beam.

The shearing force and bending moment diagrams can now be drawn as in the previous example. In Figure 40.12, let AB represent the length of the beam (*l*) drawn to scale. At A the shearing force is $wl/2$ upwards and this is shown on the graph by the ordinate AC. Because the weight of the beam is evenly distributed throughout its length, the shearing force decreases uniformly from A towards B.

At the mid-point (O) of the beam there is a shearing force of $wl/2$ downwards (half the weight of the beam) and one of $wl/2$ upwards (the reaction at A) to consider. The resultant shearing force at O is therefore zero. Finally, at B, there is a shearing force of *wl* downwards (the weight of the beam) and $wl/2$ upwards (the reaction at A) to consider, giving a resultant shearing force of $wl/2$ downwards which is represented on the graph by the ordinate BD.

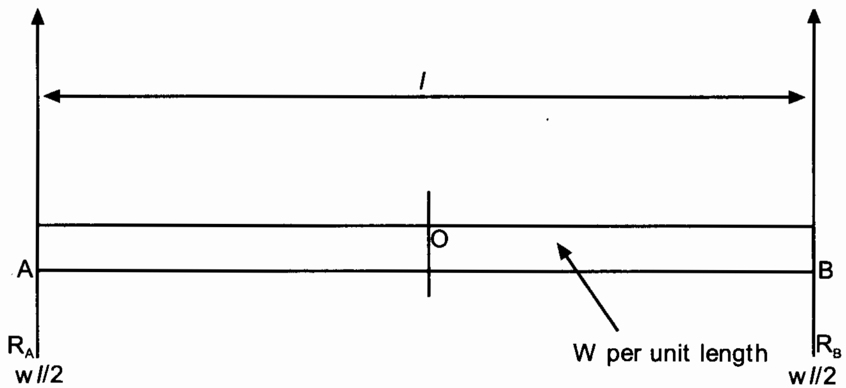


Fig. 40.12

The bending moment diagram can now be drawn in the same way or by first calculating the area under the shearing force diagram between the ordinate AC and other sections along the beam and then plotting these values as ordinates on the bending moment diagram. The bending moment diagram is represented in Figure 40.13 by the curve AEB.

It should be noted that the shearing force at any point Y which is at a distance X from the end A is given by the formula:

$$\text{Shearing force} = \frac{wl}{2} - wX$$
$$= w\left(\frac{l}{2} - X\right)$$

Also, the bending moment at Y is given by the formula:

$$\text{Bending moment} = \frac{wlX}{2} - \frac{wX^2}{2}$$
$$= \frac{wX}{2}(l - X)$$

The maximum bending moment occurs at the mid-point of the beam. Using the above formula:

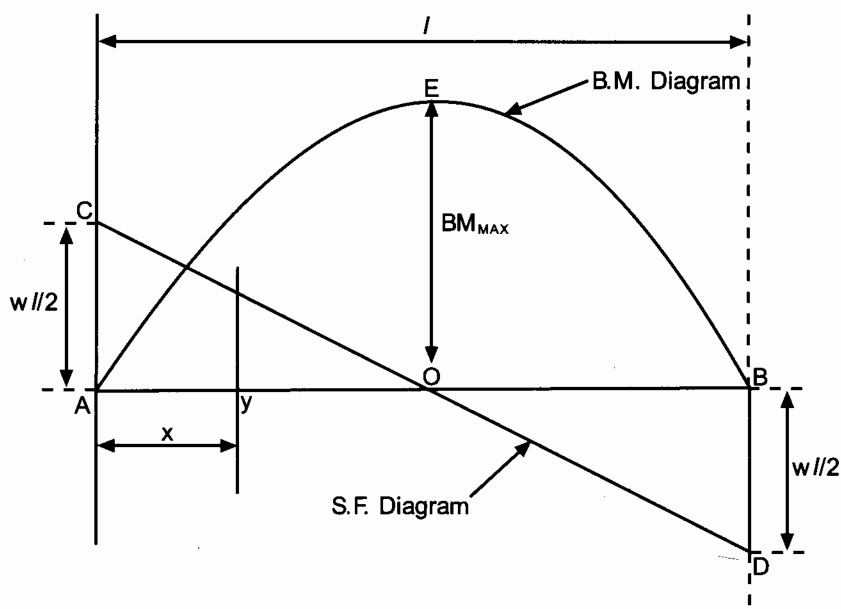


Fig. 40.13

$$\text{Bending moment} = \frac{wX}{2}(l - X)$$

$$\text{Maximum BM} = \frac{wl}{4}\left(l - \frac{1}{2}\right)$$

$$\therefore \ BM_{max} = \frac{wl^2}{8}$$

Also, the area of the shearing force diagram between the ordinate AC and O is equal to $\frac{1}{2} \times \frac{l}{2} \times \frac{wl}{2}$ or $\frac{wl^2}{8}$.

These principles can now be applied to find the shearing forces and bending moments for any simply supported beam.

Note again how BM_{max} occurs at point 'O', the point at which SF = 0. This must be so for equilibrium or balance of forces to exist.

Example

A uniform beam is 16 m long and has a mass of 10 kg per metre run. The beam is supported on two knife edges, each positioned 3 m from the end of the beam. Sketch the shearing force and bending moment diagrams and state where the bending moment is zero.

Mass per metre run = 10 kg

Total Mass of beam = 160 kg

The Reaction at C = The Reaction at B

= 80 kg

The Shear force at A = O

The Shear force at L.H. side of B = +30 kg

The Shear force at R.H. side of B = −50 kg

The Shear force at O = O

Bending moment at A = O

Bending moment at 1 m from A = $1 \times 10 \times \frac{1}{2}$

= 5 kg m (negative)

Bending moment at 2 m from A = $2 \times 10 \times 1$

= 20 kg m (negative)

Bending moment at 3 m from A = $3 \times 10 \times 1\frac{1}{2}$

= 45 kg m (negative)

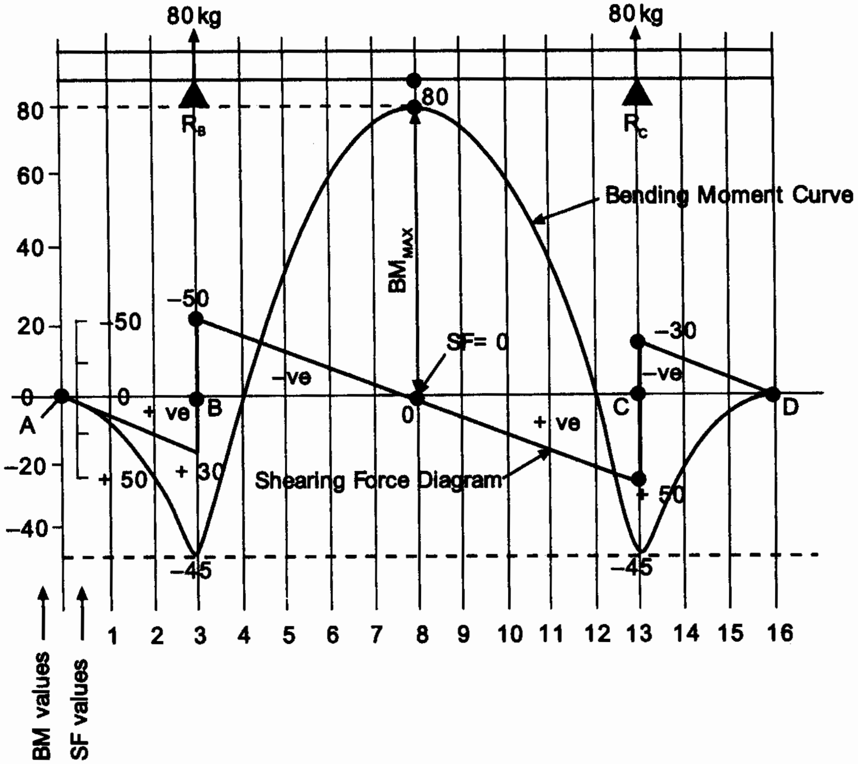


Fig. 40.14

$$\text{Bending moment at 4 m from A} = 4 \times 10 \times 2 - 80 \times 1$$
$$= 0$$
$$\text{Bending moment at 5 m from A} = 5 \times 10 \times \tfrac{5}{2} - 80 \times 2$$
$$= 35\ \text{kg m (positive)}$$
$$\text{Bending moment at 6 m from A} = 6 \times 10 \times 3 - 80 \times 3$$
$$= 60\ \text{kg m (positive)}$$
$$\text{Bending moment at 7 m from A} = 7 \times 10 \times \tfrac{7}{2} - 80 \times 4$$
$$= 75\ \text{kg m (positive)}$$
$$\text{Bending moment at 8 m from A} = 8 \times 10 \times 4 - 80 \times 5$$
$$= 80\ \text{kg m (positive)}$$

Ans. Bending moment = 0 at 4 m from each end, and at each end of the beam

The results of the above investigation into the shearing forces and consequent bending moments in simply supported beams will now be applied to find the longitudinal shearing forces and bending moments in floating vessels. Sufficient accuracy of prediction can be obtained.

However, beam theory such as this cannot be used for supertankers and ULCCs. For these very large vessels it is better to use what is known as the finite element theory. This is beyond the remit of this book.

EXERCISE 40

1 A beam AB of length 10 m is supported at each end and carries a load which increases uniformly from zero at A to 0.75 tonnes per metre run at B. Find the position and magnitude of the maximum bending moment.

2 A beam 15 m long is supported at its ends and carries two point loads. One of 5 tonnes mass is situated 6 m from one end and the other of 7 tonnes mass is 4 m from the other end. If the mass of the beam is neglected, sketch the curves of shearing force and bending moments. Also find (a), The maximum bending moment and where it occurs, and (b), The bending moment and shearing force at $\frac{1}{3}$ of the length of the beam from each end.

Chapter 41
Bending of ships

Longitudinal stresses in still water

First consider the case of a homogeneous log of rectangular section floating freely at rest in still water as shown in Figure 41.1.

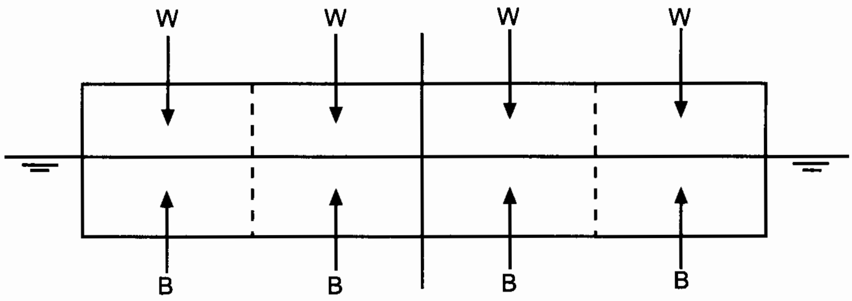


Fig. 41.1

The total weight of the log is balanced by the total force of buoyancy and the weight (W) of any section of the log is balanced by the force of buoyancy (B) provided by that section. There is therefore no bending moment longitudinally which would cause stresses to be set up in the log.

Now consider the case of a ship floating at rest in still water, on an even keel, at the light draft as shown in Figure 41.2

Although the total weight of the ship is balanced by the total force of buoyancy, neither is uniformly distributed throughout the ship's length. Imagine the ship to be cut as shown by a number of transverse sections. Imagine, too, that each section is watertight and is free to move in a vertical direction until it displaces its own weight of water. The weight of each of the end sections (1 and 5) exceeds the buoyancy which they provide and these sections will therefore sink deeper into the water until equilibrium is reached at which time each will be displacing its own weight of water. If

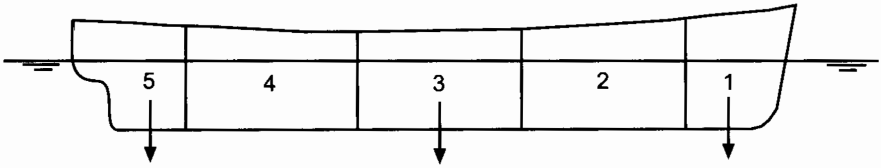


Fig. 41.2

sections 2 and 4 represent the hold sections, these are empty and they therefore provide an excess of buoyancy over weight and will rise to displace their own weight of water. If section 3 represents the engine room then, although a considerable amount of buoyancy is provided by the section, the weight of the engines and other apparatus in the engine room, may exceed the buoyancy and this section will sink deeper into the water. The nett result would be as shown in Figure 41.3 where each of the sections is displacing its own weight of water.

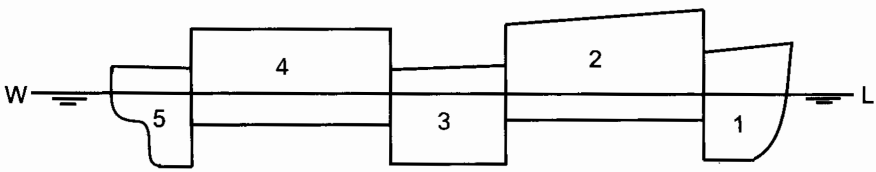


Fig. 41.3

Although the sections in the ship are not free to move in this way, bending moments, and consequently longitudinal stresses, are created by the variation in the longitudinal distribution of weight and buoyancy and these must be allowed for in the construction of the ship.

Longitudinal stresses in waves

When a ship encounters waves at sea the stresses created differ greatly from those created in still water. The maximum stresses are considered to exist when the wave length is equal to the ship's length and either a wave crest or trough is situated amidships.

Consider first the effect when the ship is supported by a wave having its crest amidships and its troughs at the bow and the stern, as shown in Figure 41.4.

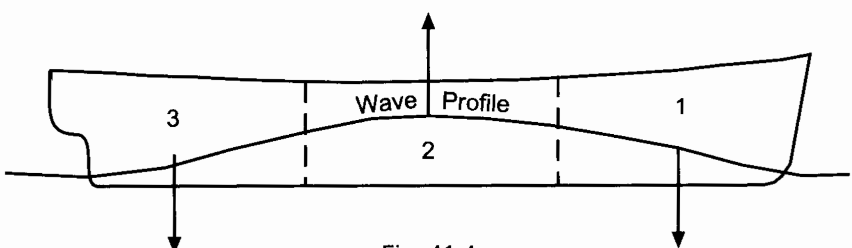


Fig. 41.4

In this case, although once more the total weight of the ship is balanced by the total buoyancy, there is an excess of buoyancy over the weight amidships and an excess of weight over buoyancy at the bow and the stern. This situation creates a tendency for the ends of the ship to move downwards and the section amidships to move upwards as shown in Figure 41.5.

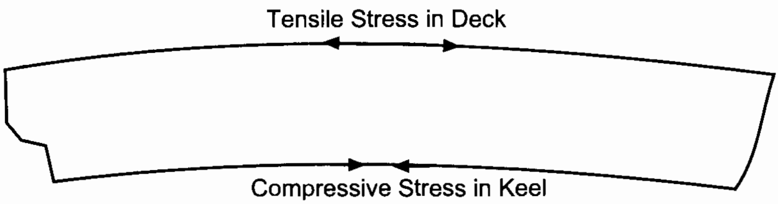


Fig. 41.5

Under these conditions the ship is said to be subjected to a 'Hogging' stress.

A similar stress can be produced in a beam by simply supporting it at its mid-point and loading each end as shown in Figure 41.6.

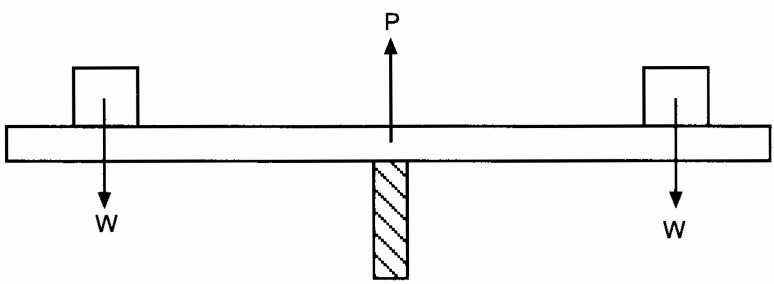


Fig. 41.6

Consider the effect after the wave crest has moved onwards and the ship is now supported by wave crests at the bow and the stern and a trough amidships as shown in Figure 41.7.

There is now an excess of buoyancy over weight at the ends and an excess of weight over buoyancy amidships. The situation creates a tendency for the bow and the stern to move upwards and the section amidships to move downwards as shown in Figure 41.8.

Under these conditions a ship is said to be subjected to a sagging stress. A stress similar to this can be produced in a beam when it is simply supported at its ends and is loaded at the mid-length as shown in Figure 41.9.

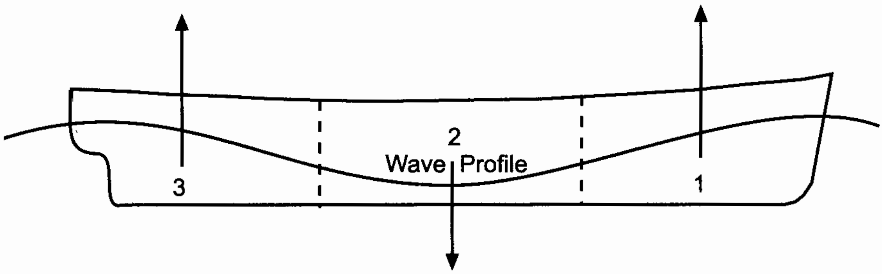


Fig. 41.7

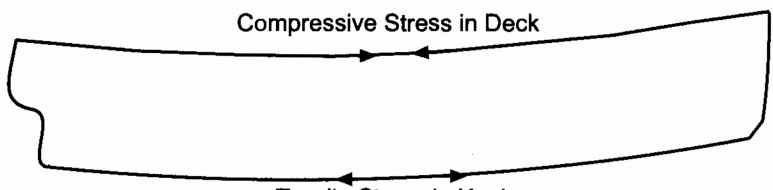


Fig. 41.8

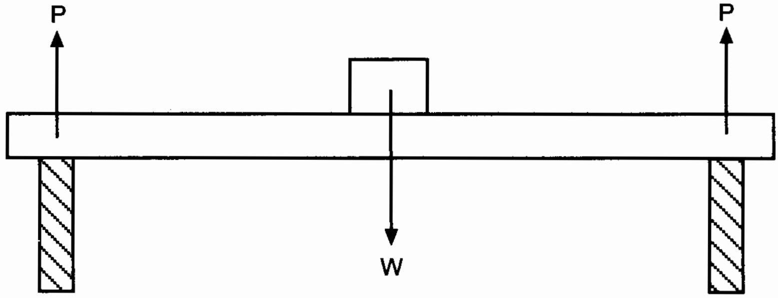


Fig. 41.9

Weight, buoyancy and load diagrams

It has already been shown that the total weight of a ship is balanced by the total buoyancy and that neither the weight nor the buoyancy is evenly distributed throughout the length of the ship.

In still water, the uneven loading which occurs throughout the length of a ship varies considerably with different conditions of loading and leads to longitudinal bending moments which may reach very high values. Care is therefore necessary when loading or ballasting a ship to keep these values within acceptable limits.

In waves, additional bending moments are created, these being brought about by the uneven distribution of buoyancy. The maximum bending moment due to this cause is considered to be created when the ship is moving head-on to waves whose length is the same as that of the ship, and when there is either a wave crest or trough situated amidships.

To calculate the bending moments and consequent shearing stresses created in a ship subjected to longitudinal bending it is first necessary to construct diagrams showing the longitudinal distribution of weight and buoyancy.

The weight diagram

A weight diagram shows the longitudinal distribution of weight. It can be constructed by first drawing a base line to represent the length of the ship, and then dividing the base line into a number of sections by equally spaced ordinates as shown in Figure 41.10. The weight of the ship between each pair of ordinates is then calculated and plotted on the diagram. In the case considered it is assumed that the weight is evenly distributed between successive ordinates but is of varying magnitude.

Let

$$CSA = \text{Cross Sectional Area}$$

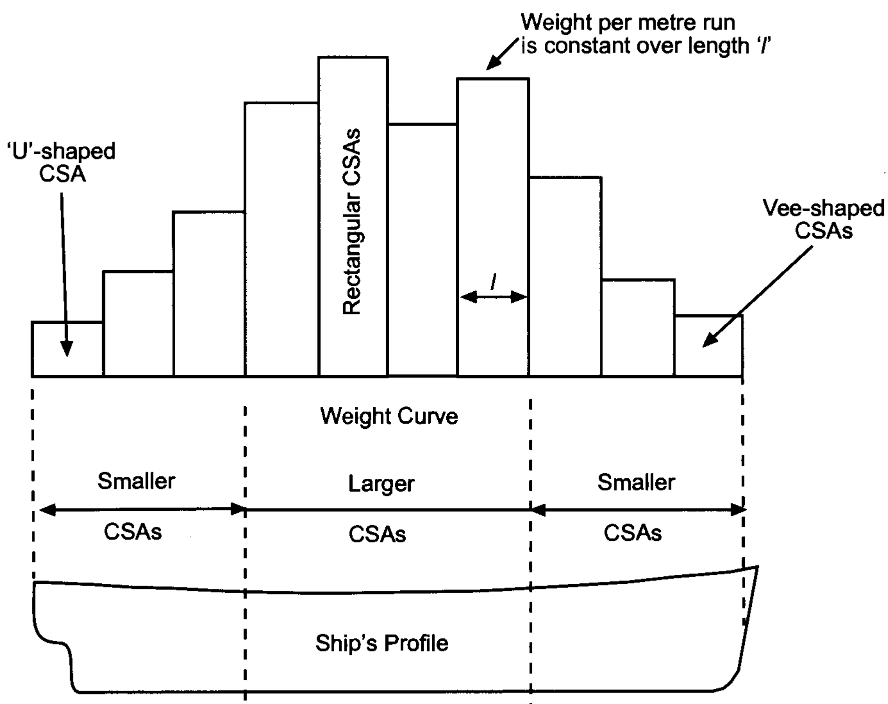


Fig. 41.10. Shows the ship divided into 10 elemental strips along her length LOA. In practice the Naval Architect may split the ship into 40 elemental strips in order to obtain greater accuracy of prediction for the weight distribution.

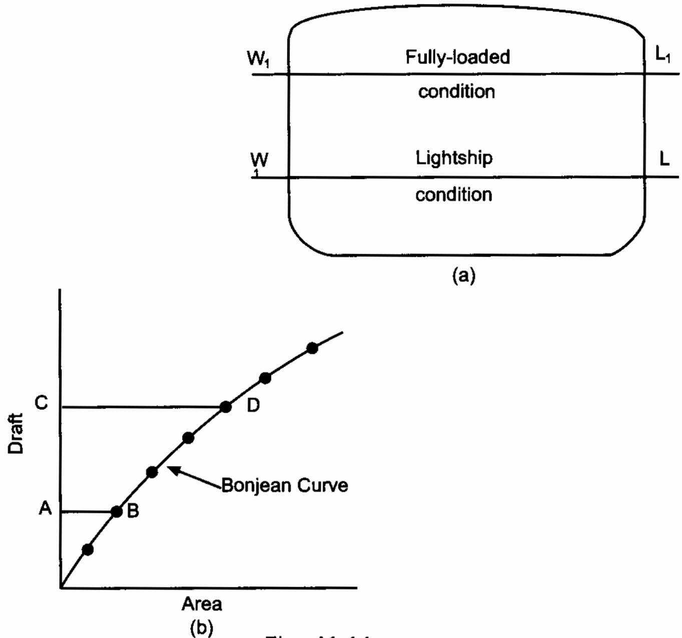


Fig. 41.11

Bonjean Curves

Bonjean Curves are drawn to give the immersed area of transverse sections to any draft and may be used to determine the longitudinal distribution of buoyancy. For example, Figure 41.11(a) shows a transverse section of a ship and Figure 41.11(b) shows the Bonjean Curve for the same section. The immersed area to the waterline WL is represented on the Bonjean Curve by ordinate AB, and the immersed area to waterline W_1L_1 is represented by ordinate CD.

In Figure 41.12 the Bonjean Curves are shown for each section throughout the length of the ship. If a wave formation is superimposed on the Bonjean Curves and adjusted until the total buoyancy is equal to the total weight of the ship, the immersed transverse area at each section can then be found by inspection and the buoyancy in tonnes per metre run is equal to the immersed area multiplied by 1.025.

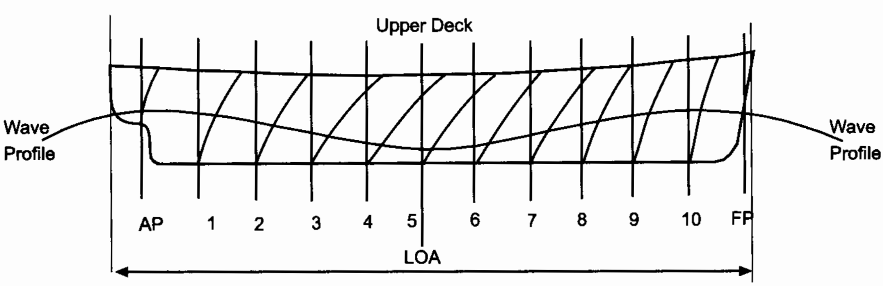


Fig. 41.12

Chapter 42
Strength curves for ships

Strength curves consist of five curves that are closely inter-related. The curves are:

1 Weight curve – tonnes/m run or kg/m run.
2 Buoyancy curve – either for hogging or sagging condition – tonnes/m or kg/m run.
3 Load curve – tonnes/m run or kg/m run.
4 Shear force curve – tonnes or kg.
5 Bending moment curve – tonnes m or kg m.

Some forms use units of MN/m run, MN and MN. m.

Buoyancy curves

A buoyancy curve shows the longitudinal distribution of buoyancy and can be constructed for any wave formation using the Bonjean Curves in the manner previously described in Chapter 41. In Figure 42.1 the buoyancy

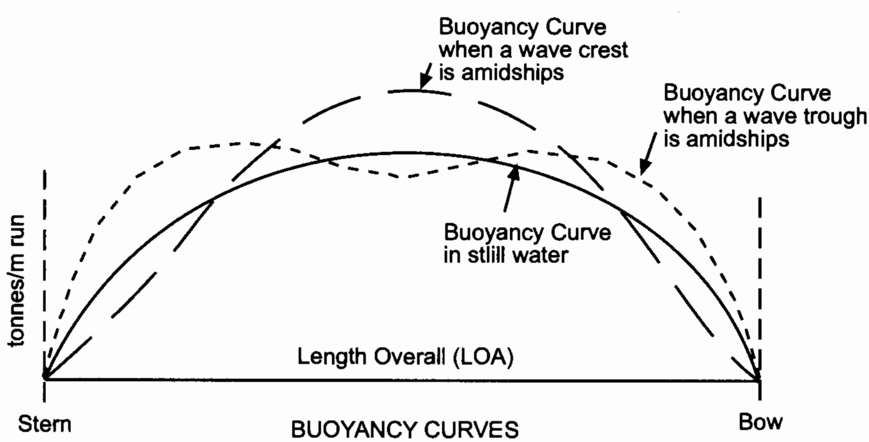


Fig. 42.1

curves for a ship are shown for the still water condition and for the conditions of maximum hogging and sagging. It should be noted that the total area under each curve is the same, i.e. the total buoyancy is the same. Units usually tonnes/m run along the length of the ship.

Load curves

A load curve shows the difference between the weight ordinate and buoyancy ordinate of each section throughout the length of the ship. The curve is drawn as a series of rectangles, the heights of which are obtained by drawing the buoyancy curve (as shown in Figure 42.1) parallel to the weight curve (as shown in Figure 41.10) at the mid-ordinate of a section and measuring the difference between the two curves. Thus the load is considered to be constant over the length of each section. An excess of weight over buoyancy is considered to produce a positive load whilst an excess of buoyancy over weight is considered to produce a negative load. Units are tonnes/m run longitudinally.

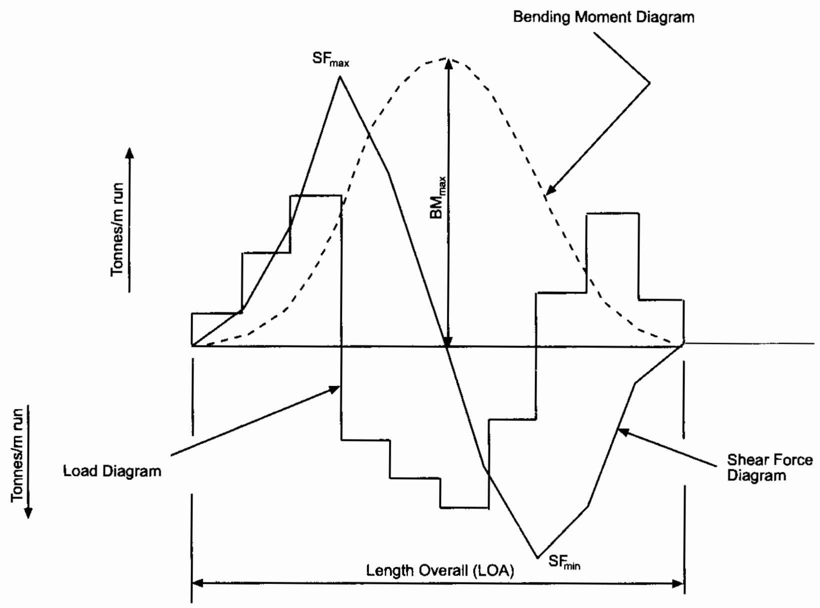


Fig. 42.2. Showing three ship strength curves for a ship in still water conditions

Shear forces and bending moments of ships

The shear force and bending moment at any section in a ship may be determined from load curve. It has already been shown that the shearing force at any section in a girder is the algebraic sum of the loads acting on either side of the section and that the bending moment acting at any section of the girder is the algebraic sum of the moments acting on either side of the section. It has also been shown that the shearing force at any section is equal to the area under the load curve from one end to the section concerned and that the bending moment at that section is equal to the area under the shearing force curve measured from the same end to that section.

Thus, for the mathematically minded, the shear force curve is the first-order integral curve of the load curve and the bending moment curve is the first-order integral curve of the shearing force curve. Therefore, the bending moment curve is the second-order integral curve of the load curve.

Figure 42.2 shows typical curves of load, shearing force and bending moments for a ship in still water.

After the still water curves have been drawn for a ship, the changes in the distribution of the buoyancy to allow for the conditions of hogging and sagging can be determined and so the resultant shearing force and bending moment curves may be found for the ship in waves.

Example

A box-shaped barge of uniform construction is 32 m long and displaces 352 tonnes when empty, is divided by transverse bulkheads into four equal compartments. Cargo is loaded into each compartment and level stowed as follows:

No. 1 hold – 192 tonnes   No. 2 hold – 224 tonnes

No. 3 hold – 272 tonnes   No. 4 hold – 176 tonnes

Construct load and shearing force diagrams, before calculating the bending moments at the bulkheads and at the position of maximum value; hence draw the bending moment diagram.

$$\text{Mass of barge per metre run} = \frac{\text{Mass of barge}}{\text{Length of barge}} = \frac{352}{32} = 11 \text{ tonnes per metre run}$$

$$\text{mass of barge when empty} = 352 \text{ tonnes}$$

$$\text{Cargo} = 192 + 224 + 272 + 176 = 864 \text{ tonnes}$$

$$\text{Total mass of barge and cargo} = 352 + 864 = 1216 \text{ tonnes}$$

$$\text{Buoyancy per metre run} = \frac{\text{Total buoyancy}}{\text{Length of barge}} = \frac{1216}{32} = 38 \text{ tonnes per metre run}$$

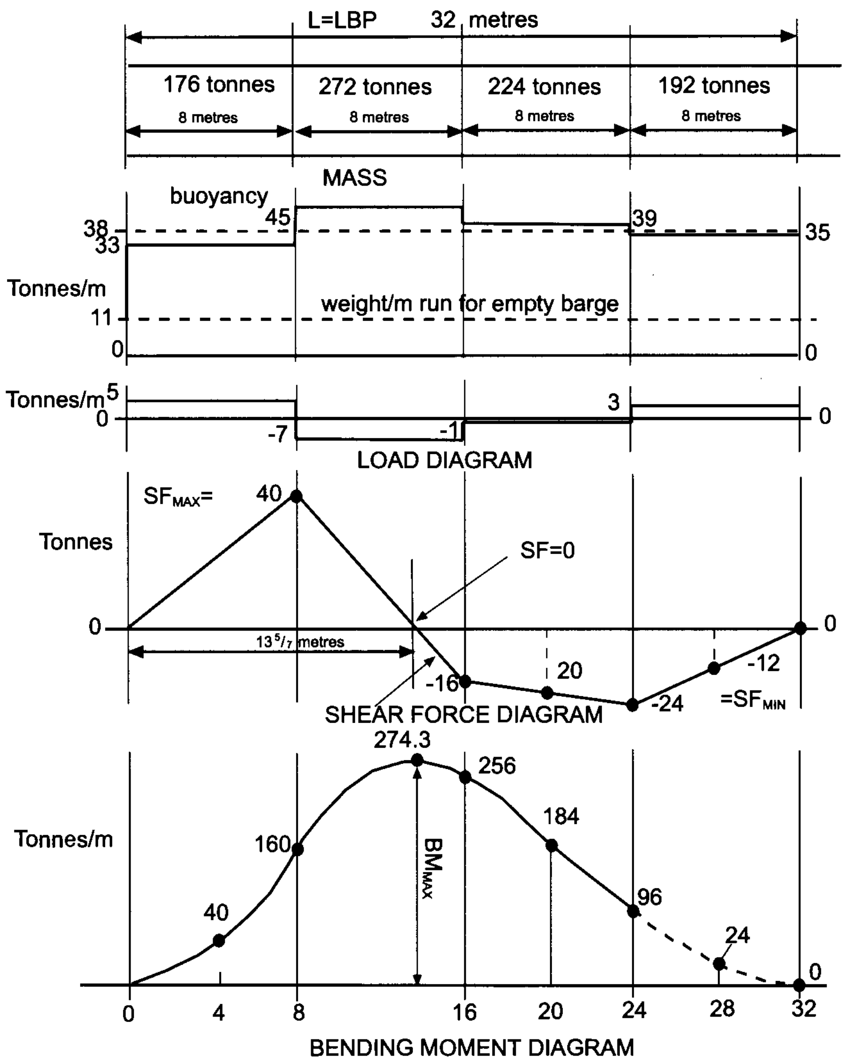


Fig. 42.3

From Figure 42.3.

Bending moments along the barge's length

$$BM_8 = \frac{8 \times 40}{2} = 160\,\text{t m}$$

$$= \underline{160\ \text{tonnes m}}$$

$$BM_0 = 0\,\text{t m}$$

$$BM_4 = \frac{20 \times 4}{2} = 40\,\text{t m}$$

$$BM_8 = \frac{8 \times 40}{2} = 160\,\text{t m}$$

$$BM_{13\frac{5}{7}} = \frac{13\frac{5}{7} \times 40}{2} = 274.3\,\text{t m}$$

$$BM_{16} = \left(\frac{13\frac{5}{7} \times 40}{2}\right) - \left(\frac{2\frac{2}{7} \times 16}{2}\right) = 256\,\text{t m}$$

$$BM_{20} = 256 - \left(\frac{16 + 20}{2}\right) \cdot 4 = 184\,\text{t m}$$

$$BM_{24} = 184 - \left(\frac{20 + 24}{2}\right) \cdot 4 = 96\,\text{t m}$$

$$BM_{28} = 96 - \left(\frac{24 + 12}{2}\right) \cdot 4 = 24\,\text{t m}$$

$$BM_{32} = 24 - \left(\frac{12 \times 4}{2}\right) = 0\,\text{t m}$$

Murray's Method

Murray's Method is used to find the total longitudinal bending moment amidships on a ship in waves and is based on the division of the total bending moment into two parts:

(a) the Still Water Bending Moment, and
(b) the wave bending moment.

The Still Water Bending Moment is the longitudinal bending moment amidships when the ship is floating in still water.

When using Murray's Method the wave bending moment amidships is that produced by the waves when the ship is supported on what is called a 'Standard Wave'. A Standard Wave is one whose length is equal to the length of the ship (L), and whose height is equal to $0.607\sqrt{L}$, where L is measured in metres. See Figure 42.4.

The Wave Bending Moment is then found using the formula:

$$\text{WBM} = b \cdot B \cdot L^{2.5} \times 10^{-3} \text{ tonnes metres}$$

where B is the beam of the ship in metres and b is a constant based on the ship's block coefficient (C_b) and on whether the ship is hogging or sagging. The value of b can be obtained from the table on page 351.

The Still Water Bending Moment (SWBM)

Let

W_F represent the moment of the weight forward of amidships,
B_F represent the moment of buoyancy forward of amidships,

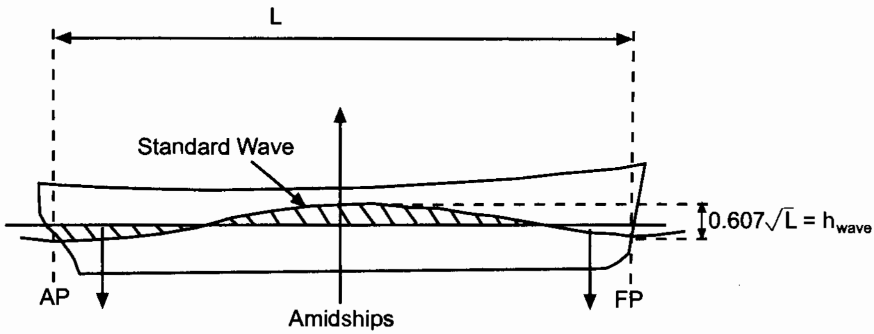


Fig. 42.4

Murray's coefficient 'b' values

	Values of b	
C_b	*Hogging*	*Sagging*
0.80	10.555	11.821
0.78	10.238	11.505
0.76	9.943	11.188
0.74	9.647	10.850
0.72	9.329	10.513
0.70	9.014	10.175
0.68	8.716	9.858
0.66	8.402	9.541
0.64	8.106	9.204
0.62	7.790	8.887
0.60	7.494	8.571

Let

W_A represent the moment of the weight aft of amidships,
B_A represent the moment of the buoyancy aft of amidships, and
W represent the ship's displacement,

then:

$$\text{Still Water Bending Moment (SWBM)} = W_F - B_F$$
$$= W_A - B_A$$

This equation can be accurately evaluated by resolving in detail the many constituent parts, but Murray's Method may be used to give an approximate solution with sufficient accuracy for practical purposes.

The following approximations are then used:

$$\text{Mean Weight Moment } (M_W) = \frac{W_F + W_A}{2}$$

This moment is calculated using the full particulars of the ship in its loaded condition.

$$\text{Mean Buoyancy Moment } (M_B) = \frac{W}{2} \times \text{Mean LCB of fore and aft bodies}$$

An analysis of a large number of ships has shown that the Mean LCB of the fore and aft bodies for a trim not exceeding 0.01 L can be found using the formula:

$$\text{Mean LCB} = L \times C$$

where L is the length of the ship in metres, and the value of C can be found from the following table in terms of the block coefficient (C_b) for the ship at a draft of 0.06 L.

Murray's coefficient 'C' values

Draft	*C*
0.06 L	$0.179C_b + 0.063$
0.05 L	$0.189C_b + 0.052$
0.04 L	$0.199C_b + 0.041$
0.03 L	$0.209C_b + 0.030$

The Still Water Bending Moment Amidships (SWBM) is then given by the formula:

$$\text{SWBM} = \text{Mean Weight Moment } (M_W) - \text{Mean Buoyancy Moment } (M_B)$$

or

$$\text{SWBM} = \frac{W_F + W_A}{2} - \frac{W}{2} \cdot L \cdot C$$

where the value of C is found from the table above.

If the Mean Weight Moment is greater than the Mean Buoyancy Moment then the ship will be hogged, but if the Mean Buoyancy Moment exceeds the Mean Weight Moment then the ship will sag. So

(i) If $M_W > M_B$ ········ ship hogs.
(ii) If $M_B > M_W$ ········ ship sags. } M_W ~ M_B

The Wave Bending Moment (WBM)

The actual wave bending moment depends upon the height and the length of the wave and the beam of the ship. If a ship is supported on a Standard

Wave, as defined above, then the Wave Bending Moment (WBM) can be calculated using the formula:

$$\text{WBM} = b \cdot B \cdot L^{2.5} \times 10^{-3} \text{ tonnes metres}$$

where B is the beam of the ship and where the value of b is found from the table on page 351.

Example

The length LBP of a ship is 200 m, the beam is 30 m and the block coefficient is 0.750. The hull weight is 5000 tonnes having LCG 25.5 m from amidships. The mean LCB of the fore and after bodies is 25 m from amidships. Values of the constant b are: hogging 9.795 and sagging 11.02.

Given the following data and using Murray's Method, calculate the longitudinal bending moments amidships for the ship on a standard wave with: (a) the crest amidships, and (b) the trough amidships. Use Figure 42.5 to obtain solution.

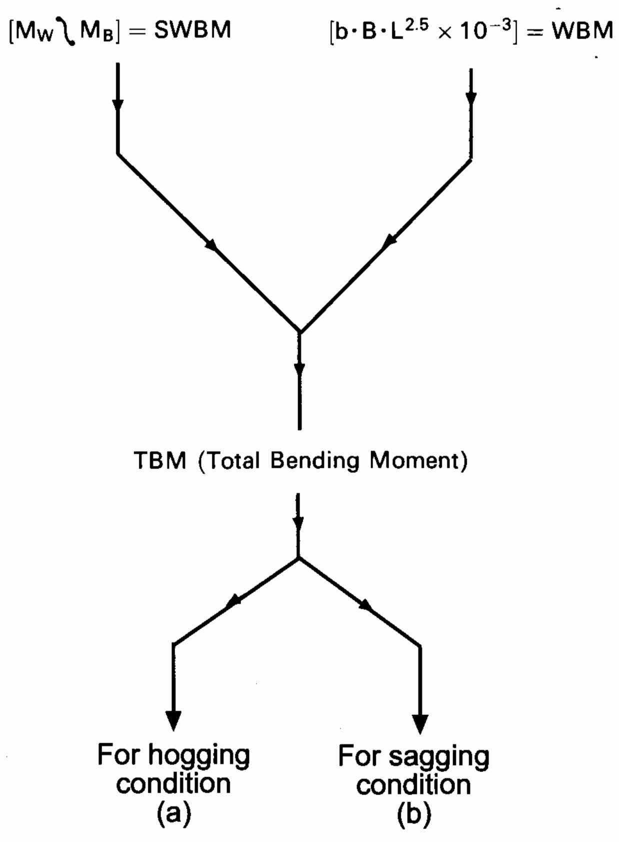


Fig. 42.5. Line diagram for solution using Murray's method.

Data

Item	*Weight*	*LCG from amidships*
No. 1 hold	1800 t	55.0 m aft
No. 2 hold	3200 t	25.5 m forward
No. 3 hold	1200 t	5.5 m forward
No. 4 hold	2200 t	24.0 m aft
No. 5 hold	1500 t	50.0 m aft
Machinery	1500 t	7.5 m aft
Fuel oil	400 t	8.0 m aft
Fresh water	150 t	10.0 m forward

Item	*Weight*	*LCG from amidships*	*Moment*
No. 1 hold	1800	55.0 m forward	99 000
No. 2 hold	3200	25.5 m forward	81 600
No. 3 hold	1200	5.5 m forward	6 600
No. 4 hold	2200	24.0 m aft	52 800
No. 5 hold	1500	50.0 m aft	75 000
Machinery	1500	7.5 m aft	11 250
Fuel oil	400	8.0 m aft	3200
Fresh water	150	10.0 m forward	1500
Hull	5000	25.5 m	127 500
	16 950		458 450

To find the Still Water Bending Moment (SWBM)

$$\text{Mean Weight Moment } (M_W) = \frac{W_F + W_A}{2}$$

$$= \frac{458\,450}{2}$$

$$\underline{M_W = 229\,225 \text{ t m}}$$

$$\text{Mean Buoyancy Moment } (M_B) = \frac{W}{2} \cdot \text{LCB} = \frac{16\,950}{2} \cdot 25$$

$$= 211\,875 \text{ t m}$$

$$\text{Still Water Bending Moment (SWBM)} = M_W - M_B$$

$$= 229\,225 - 211\,875$$

$$\text{SWBM} = 17\,330 \text{ t m (Hogging) because } M_W > M_B$$

(see page 352)

Wave Bending Moment (WBM)

$$\text{Wave Bending Moment (WBM)} = b \cdot B \cdot L^{2.5} \times 10^{-3}\ \text{t m}$$

$$\text{WBM Hogging} = 9.795 \times 30 \times 200^{2.5} \times 10^{-3}\ \text{t m}$$

$$= 166\,228\ \text{t m}$$

$$\text{WBM Sagging} = 11.02 \times 30 \times 200^{2.5} \times 10^{-3}\ \text{t m}$$

$$= 187\,017\ \text{t m}$$

Total Bending Moment (TBM)

$$\text{TBM Hogging} = \text{WBM hogging} + \text{SWBM hogging}$$

$$= 166\,228 + 17\,350$$

$$= 183\,578\ \text{t m}$$

$$\text{TBM Sagging} = \text{WBM Sagging} - \text{SWBM hogging}$$

$$= 187\,017 - 17\,350$$

$$= 169\,667\ \text{t m}$$

Answer (a) with crest amidships, the Total Bending Moment, *TBM is 183 578 tonnes metres.*

Answer (b) with trough amidships, the Total Bending Moment, *TBM is 169 667 tonnes metres.*

The *greatest* danger for a ship to break her back is when the wave crest is at amidships, or when the wave trough is at amidships with the crests at the stem and at the bow.

In the previous example the greatest BM occurs with the crest amidships. Consequently, this ship would fracture across the Upper Deck if the tensile stress due to hogging condition became too high.