Chapter 4
Laws of flotation

Archimedes' Principle states that when a body is wholly or partially immersed in a fluid it appears to suffer a loss in mass equal to the mass of the fluid it displaces.

The mass density of fresh water is 1000 kg per cu. m. Therefore, when a body is immersed in fresh water it will appear to suffer a loss in mass of 1000 kg for every 1 cu. m of water it displaces.

When a box measuring 1 cu. m and of 4000 kg mass is immersed in fresh water it will appear to suffer a loss in mass of 1000 kg. If suspended from a spring balance the balance would indicate a mass of 3000 kg.

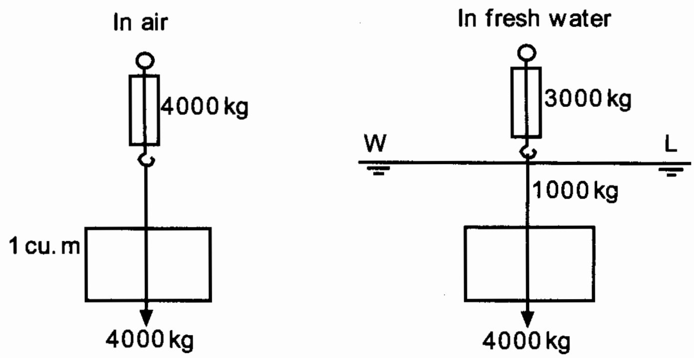

Fig. 4.1

Since the actual mass of the box is not changed, there must be a force acting vertically upwards to create the apparent loss of mass of 1000 kg. This force is called the *force of buoyancy*, and is considered to act vertically upwards through a point called the *centre of buoyancy*. The centre of buoyancy is the centre of gravity of the underwater volume.

Now consider the box shown in Figure 4.2(a) which also has a mass of 4000 kg, but has a volume of 8 cu. m. If totally immersed in fresh water it will displace 8 cu. m of water, and since 8 cu. m of fresh water has a mass of

8000 kg, there will be an upthrust or force of buoyancy causing an apparent loss of mass of 8000 kg. The resultant apparent loss of mass is 4000 kg. When released, the box will rise until a state of equilibrium is reached, i.e. when the buoyancy is equal to the mass of the box. To make the buoyancy produce a loss of mass of 4000 kg the box must be displacing 4 cu m of water. This will occur when the box is floating with half its volume immersed, and the resultant force then acting on the box will be zero. This is shown in Figure 4.2(c).

Now consider the box to be floating in fresh water with half its volume immersed as shown in Figure 4.2(c). If a mass of 1000 kg be loaded on deck as shown in Figure 4.3(a) the new mass of the body will be 5000 kg, and since this exceeds the buoyancy by 1000 kg, it will move downwards.

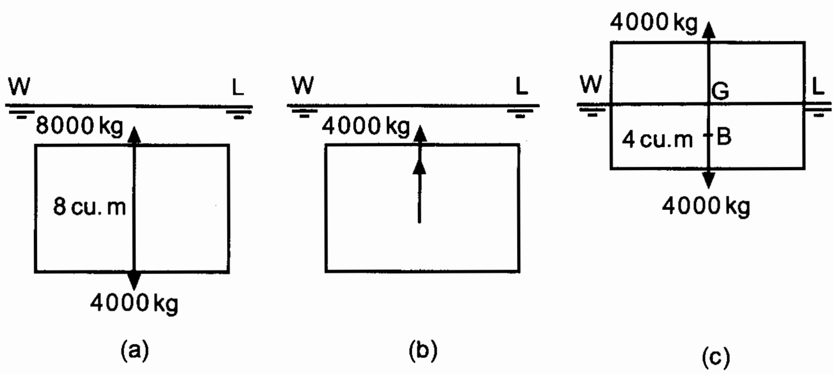

Fig. 4.2

The downwards motion will continue until buoyancy is equal to the mass of the body. This will occur when the box is displacing 5 cu. m of water and the buoyancy is 5000 kg, as shown in Figure 4.3(b).

The conclusion which may be reached from the above is that for a body to float at rest in still water, it must be displacing its own weight of water and the centre of gravity must be vertically above or below the centre of buoyancy.

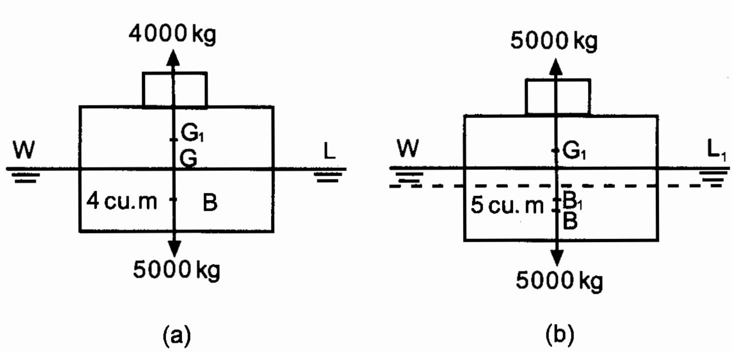

Fig. 4.3

The variable immersion hydrometer

The variable immersion hydrometer is an instrument, based on the Law of Archimedes, which is used to determine the density of liquids. The type of hydrometer used to find the density of the water in which a ship floats is usually made of a non-corrosive material and consists of a weighted bulb with a narrow rectangular stem which carries a scale for measuring densities between 1000 and 1025 kilograms per cubic metre, i.e. 1.000 and 1.025 t/m^3.

The position of the marks on the stem are found as follows. First let the hydrometer, shown in Figure 4.4, float upright in fresh water at the mark X. Take the hydrometer out of the water and weigh it. Let the mass be M_x kilograms. Now replace the hydrometer in fresh water and add lead shot in the bulb until it floats with the mark Y, at the upper end of the stem, in the

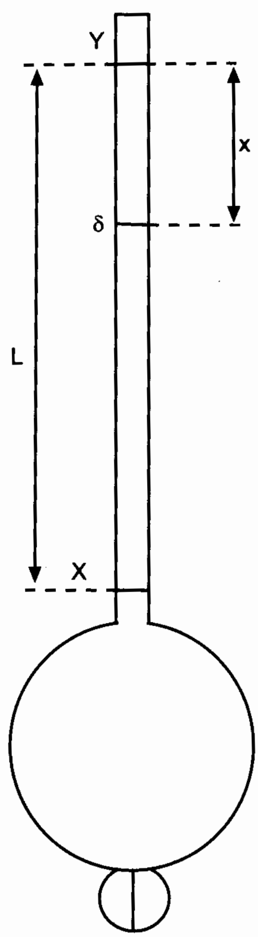

Fig. 4.4

waterline. Weigh the hydrometer again and let its mass now be M_y kilograms.

The mass of water displaced by the stem between X and Y is therefore equal to $M_y - M_x$ kilograms. Since 1000 kilograms of fresh water occupy one cubic metre, the volume of the stem between X and Y is equal to $\frac{M_y - M_x}{1000}$ cu. m.

Let L represent the length of the stem between X and Y, and let 'a' represent the cross-sectional area of the stem.

$$a = \frac{\text{Volume}}{\text{Length}}$$

$$= \frac{M_y - M_x}{1000\,L} \text{ sq m}$$

Now let the hydrometer float in water of density $\delta\,\text{kg/m}^3$ with the waterline 'x' metres below Y.

$$\text{Volume of water displaced} = \frac{M_y}{1000} - x\,a$$

$$= \frac{M_y}{1000} - x\left(\frac{M_y - M_x}{1000\,L}\right) \qquad \text{(I)}$$

But

$$\text{Volume of water displaced} = \frac{\text{Mass of water displaced}}{\text{Density of water displaced}}$$

$$= \frac{M_y}{1000\,\delta} \qquad \text{(II)}$$

$$\text{Equate (I) and (II)} \therefore \frac{M_y}{1000\,\delta} = \frac{M_y}{1000} - x\left(\frac{M_y - M_x}{1000\,L}\right)$$

or

$$\delta = \frac{M_y}{M_y - x\left(\frac{M_y - M_x}{L}\right)}$$

In this equation, M_y, M_x and L are known constants whilst δ and x are variables. Therefore, to mark the scale it is now only necessary to select various values of δ and to calculate the corresponding values of x.

Tonnes per Centimetre Immersion (TPC)

The TPC for any draft is the mass which must be loaded or discharged to change a ship's mean draft in salt water by one centimetre, where

$$\text{TPC} = \frac{\text{water-plane area}}{100} \times \text{density of water}$$

$$\therefore \text{TPC} = \frac{\text{WPA}}{100} \times \rho$$

WPA is in m^2.
ρ is in t/m^3.

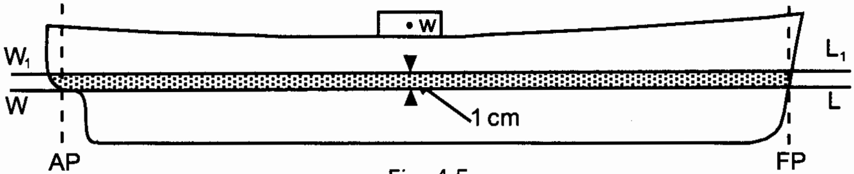

Fig. 4.5

Consider a ship floating in *salt water* at the waterline WL as shown in Figure 4.5. Let 'A' be the area of the water-plane in square metres.

Now let a mass of 'w' tonnes be loaded so that the mean draft is increased by one centimetre. The ship then floats at the waterline W_1 L_1. Since the draft has been increased by one centimetre, the mass loaded is equal to the TPC for this draft. Also, since an extra mass of water equal to the mass loaded must be displaced, then the mass of water in the layer between WL and W_1 L_1 is also equal to the TPC.

$$\text{Mass} = \text{Volume} \times \text{Density}$$

$$= A \times \frac{1}{100} \times \frac{1025}{1000} \text{ tonnes} = \frac{1.025\,A}{100} \text{ tonnes}$$

$$\therefore \text{TPC}_{sw} = \frac{1.025\,A}{100} = \frac{\text{WPA}}{97.56}. \quad \text{Also, } \text{TPC}_{fw} = \frac{\text{WPA}}{100}$$

TPC in dock water

Note. When a ship is floating in dock water of a relative density other than 1.025 the weight to be loaded or discharged to change the mean draft by 1 centimetre (TPC_{dw}) may be found from the TPC in salt water (TPC_{sw}) by simple proportion as follows:

$$\frac{\text{TPC}_{dw}}{\text{TPC}_{sw}} = \frac{\text{relative density of dock water } (\text{RD}_{dw})}{\text{relative density of salt water } (\text{RD}_{sw})}$$

or

$$\text{TPC}_{dw} = \frac{\text{RD}_{dw}}{1.025} \times \text{TPC}_{sw}$$

Reserve buoyancy

It has already been shown that a floating vessel must displace its own weight of water. Therefore, it is the submerged portion of a floating vessel which provides the buoyancy. The volume of the enclosed spaces above the waterline are not providing buoyancy but are being held in reserve. If extra weights are loaded to increase the displacement, these spaces above the waterline are there to provide the extra buoyancy required. Thus, *reserve buoyancy* may be defined as the volume of the enclosed spaces above the waterline. It may be expressed as a volume or as a percentage of the total volume of the vessel.

Example 1

A box-shaped vessel 105 m long, 30 m beam, and 20 m deep, is floating upright in fresh water. If the displacement is 19 500 tonnes, find the volume of reserve buoyancy.

$$\text{Volume of water displaced} = \frac{\text{Mass}}{\text{Density}} = 19\,500\ \text{cu. m}$$

$$\text{Volume of vessel} = 105 \times 30 \times 20\ \text{cu. m}$$

$$= 63\,000\ \text{cu. m}$$

$$\text{Reserve buoyancy} = \text{Volume of vessel} - \text{volume of water displaced}$$

Ans. Reserve buoyancy = 43 500 cu. m

Example 2

A box-shaped barge 16 m × 6 m × 5 m is floating alongside a ship in fresh water at a mean draft of 3.5 m. The barge is to be lifted out of the water and loaded on to the ship with a heavy-lift derrick. Find the load in tonnes borne by the purchase when the draft of the barge has been reduced to 2 metres.

Note. By Archimedes' Principle the barge suffers a loss in mass equal to the mass of water displaced. The mass borne by the purchase will be the difference between the actual mass of the barge and the mass of water displaced at any draft, or the difference between the mass of water originally displaced by the barge and the new mass of water displaced.

$$\text{Mass of the barge} = \text{Original mass of water displaced}$$

$$= \text{Volume} \times \text{density}$$

$$= 16 \times 6 \times 3.5 \times 1\ \text{tonnes}$$

$$\text{Mass of water displace at 2 m draft} = 16 \times 6 \times 2 \times 1\ \text{tonnes}$$

$$\therefore \text{Load borne by the purchase} = 16 \times 6 \times 1 \times (3.5 - 2)\ \text{tonnes}$$

Ans. = 144 tonnes

Example 3

A cylindrical drum 1.5 m long and 60 cm in diameter has mass 20 kg when empty. Find its draft in water of density 1024 kg per cu. m if it contains 200 litres of paraffin of relative density 0.6, and is floating with its axis perpendicular to the waterline (Figure 4.6).

Note. The drum must displace a mass of water equal to the mass of the drum plus the mass of the paraffin.

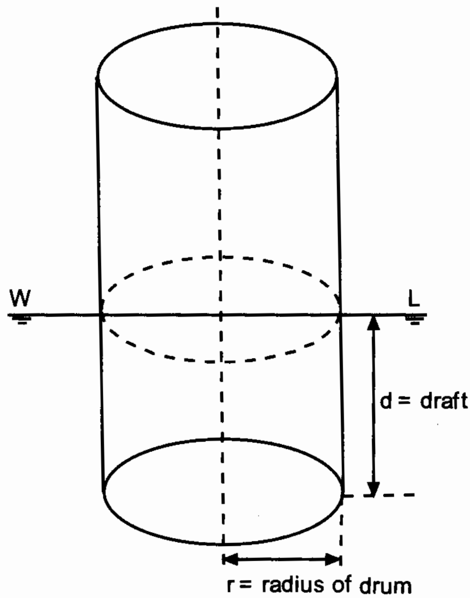

Fig. 4.6

$$\text{Density of the paraffin} = \text{SG} \times 1000\,\text{kg per cu. m}$$
$$= 600\,\text{kg per cu. m}$$
$$\text{Mass of the paraffin} = \text{Volume} \times \text{density} = 0.2 \times 600\,\text{kg}$$
$$= 120\,\text{kg}$$
$$\text{Mass of the drum} = 20\,\text{kg}$$
$$\text{Total mass} = 140\,\text{kg}$$

Therefore the drum must displace 140 kg of water.

$$\text{Volume of water displaced} = \frac{\text{Mass}}{\text{Density}} = \frac{140}{1024}\ \text{cu. m}$$

$$\text{Volume of water displaced} = 0.137\ \text{cu. m}$$

Let d = draft, and r = radius of the drum, where $r = \frac{60}{2} = 30\,\text{cm} = 0.3\,\text{m}$.

$$\text{Volume of water displaced (V)} = \pi r^2 d$$

or

$$d = \frac{V}{\pi r^2}$$

$$= \frac{0.137}{\frac{22}{7} \times 0.3 \times 0.3} \text{ m}$$

$$= 0.484 \text{ m}$$

Ans. Draft = 0.484 m

Homogeneous logs of rectangular section

The draft at which a rectangular homogeneous log will float may be found as follows:

$$\text{Mass of log} = \text{Volume} \times \text{density}$$

$$= L \times B \times D \times \text{SG of log} \times 1000 \text{ kg}$$

$$\text{Mass of water displaced} = \text{Volume} \times \text{density}$$

$$= L \times B \times d \times \text{SG of water} \times 1000 \text{ kg}$$

$$\text{But Mass of water displaced} = \text{Mass of log}$$

$$\therefore L \times B \times d \times \text{SG of water} \times 1000 = L \times B \times D \times \text{SG of log} \times 1000$$

or

$$d \times \text{SG of water} = D \times \text{SG of log}$$

$$\frac{\text{Draft}}{\text{Depth}} = \frac{\text{SG of log}}{\text{SG of water}} \text{ or } \frac{\text{relative density of log}}{\text{relative density of water}}$$

Example 4

Find the distance between the centres of gravity and buoyancy of a rectangular log 1.2 m wide, 0.6 m deep, and of relative density 0.8 when floating in fresh water with two of its sides parallel to the waterline.

If BM is equal to $\frac{b^2}{12\,d}$ determine if this log will float with two of its sides parallel to the waterline.

Note. The centre of gravity of a homogeneous log is at its geometrical centre. See Figure 4.7

$$\frac{\text{Draft}}{\text{Depth}} = \frac{\text{Relative density of log}}{\text{Relative density of water}}$$

$$\text{Draft} = \frac{0.6 \times 0.8}{1}$$

$$\left.\begin{aligned} \text{Draft} &= 0.48 \text{ m} \\ KB &= 0.24 \text{ m} \\ KG &= 0.30 \text{ m} \end{aligned}\right\} \text{see Figure 4.8}$$

Ans. BG = 0.06 m

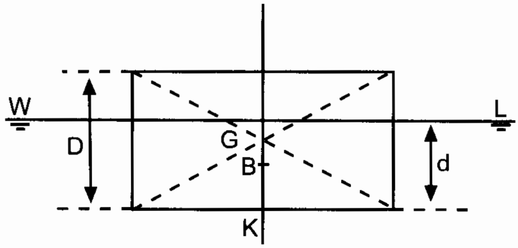

Fig. 4.7

$$BM = \frac{b^2}{12 \times d} = \frac{1.2^2}{12 \times 0.48} = 0.25 \text{ m}$$

$$\text{KB as above} = +0.24 \text{ m}$$

$$KM = KB + BM = +0.49 \text{ m}$$

$$-KG = -0.30 \text{ m}$$

$$GM = 0.19 \text{ m}$$

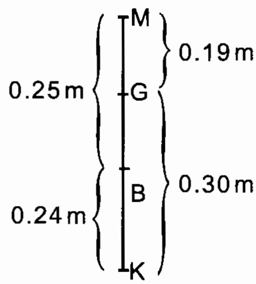

Fig. 4.8

Conclusion

Because G is below M, this homogeneous log is in stable equilibrium. Consequently, it *will* float with two of its sides parallel to the waterline.

Exercise 4

1 A drum of mass 14 kg when empty, is 75 cm long, and 60 cm in diameter. Find its draft in salt water if it contains 200 litres of paraffin of relative density 0.63.

2 A cube of wood of relative density 0.81 has sides 30 cm long. If a mass of 2 kg is placed on the top of the cube with its centre of gravity vertically over that of the cube, find the draft in salt water.

3 A rectangular tank (3 m × 1.2 m × 0.6 m) has no lid and is floating in fresh water at a draft of 15 cm. Calculate the minimum amount of fresh water which must be poured into the tank to sink it.

4 A cylindrical salvage buoy is 5 metres long, 2.4 metres in diameter, and floats on an even keel in salt water with its axis in the water-plane. Find the upthrust which this buoy will produce when fully immersed.

5 A homogeneous log of rectangular cross-section is 30 cm wide and 25 cm deep. The log floats at a draft of 17 cm. Find the reserve buoyancy and the distance between the centre of buoyancy and the centre of gravity.

6 A homogeneous log of rectangular cross-section is 5 m. long, 60 cm wide, 40 cm deep, and floats in fresh water at a draft of 30 cm. Find the mass of the log and its relative density.

7 A homogeneous log is 3 m long, 60 cm wide, 60 cm deep, and has relative density 0.9. Find the distance between the centres of buoyancy and gravity when the log is floating in fresh water.

8 A log of square section is 5 m × 1 m × 1 m. The relative density of the log is 0.51 and it floats half submerged in dock water. Find the relative density of the dock water.

9 A box-shaped vessel 20 m × 6 m × 2.5 m floats at a draft of 1.5 m in water of density 1013 kg per cu. m. Find the displacement in tonnes, and the height of the centre of buoyancy above the keel.

10 An empty cylindrical drum 1 metre long and 0.6 m. in diameter has mass 20 kg. Find the mass which must be placed in it so that it will float with half of its volume immersed in (a) salt water, and (b) fresh water.

11 A lifeboat, when fully laden, displaces 7.2 tonnes. Its dimensions are 7.5 m × 2.5 m × 1 m, and its block coefficient 0.6. Find the percentage of its volume under water when floating in fresh water.

12 A homogeneous log of relative density 0.81 is 3 metres long, 0.5 metres square cross-section, and is floating in fresh water. Find the displacement of the log, and the distance between the centres of gravity and buoyancy.

13 A box-shaped barge 55 m × 10 m × 6 m. is floating in fresh water on an even keel at 1.5 m draft. If 1800 tonnes of cargo is now loaded, find the difference in the height of the centre of buoyancy above the keel.

14 A box-shaped barge 75 m × 6 m × 4 m displaces 180 tonnes when light. If

360 tonnes of iron are loaded while the barge is floating in fresh water, find her final draft and reserve buoyancy.

15 A drum 60 cm in diameter and 1 metre long has mass 30 kg when empty. If this drum is filled with oil of relative density 0.8, and is floating in fresh water, find the percentage reserve buoyancy.

Chapter 5
Effect of density on draft and displacement

Effect of change of density when the displacement is constant

When a ship moves from water of one density to water of another density, without there being a change in her mass, the draft will change. This will happen because the ship must displace the same mass of water in each case. Since the density of the water has changed, the volume of water displaced must also change. This can be seen from the formula:

$$\text{Mass} = \text{Volume} \times \text{Density}$$

If the density of the water increases, then the volume of water displaced must decrease to keep the mass of water displaced constant, and vice versa.

The effect on box-shaped vessels

$$\text{New mass of water displaced} = \text{Old mass of water displaced}$$

$$\therefore \text{New volume} \times \text{new density} = \text{Old volume} \times \text{Old density}$$

$$\frac{\text{New volume}}{\text{Old volume}} = \frac{\text{Old density}}{\text{New density}}$$

$$\text{But volume} = \text{L} \times \text{B} \times \text{draft}$$

$$\therefore \frac{\text{L} \times \text{B} \times \text{New draft}}{\text{L} \times \text{B} \times \text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

or

$$\frac{\text{New draft}}{\text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

Example 1

A box-shaped vessel floats at a mean draft of 2.1 metres, in dock water of density 1020 kg per cu. m. Find the mean draft for the same mass displacement in salt water of density 1025 kg per cubic metre.

$$\frac{\text{New draft}}{\text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

$$\text{New draft} = \frac{\text{Old density}}{\text{New density}} \times \text{Old draft}$$

$$= \frac{1020}{1025} \times 2.1\,\text{m}$$

$$= 2.09\,\text{m}$$

Ans. New draft = 2.09 m

Example 2

A box-shaped vessel floats upright on an even keel as shown in fresh water of density 1000 kg per cu. m, and the centre of buoyancy is 0.50 m above the keel. Find the height of the centre of buoyancy above the keel when the vessel is floating in salt water of density 1025 kg per cubic metre.

Note. The centre of buoyancy is the geometric centre of the underwater volume and for a box-shaped vessel must be at half draft, i.e. KB = $\frac{1}{2}$ draft.

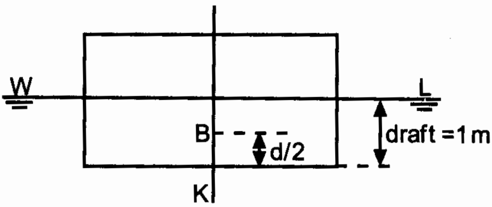

Fig. 5.1

In Fresh Water

KB = 0.5 m, and since KB = $\frac{1}{2}$ draft, then draft = 1 m

In Salt Water

$$\frac{\text{New draft}}{\text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

$$\text{New draft} = \text{Old draft} \times \frac{\text{Old density}}{\text{New density}}$$

$$= 1 \times \frac{1000}{1025}$$

$$\text{New draft} = 0.976\,\text{m}$$

$$\text{New KB} = \tfrac{1}{2}\text{ new draft}$$

Ans. New KB = 0.488 m, say 0.49 m.

The effect on ship-shaped vessels

It has already been shown that when the density of the water in which a vessel floats is changed the draft will change, but the mass of water in kg or tonnes displaced will be unchanged. i.e.

$$\text{New displacement} = \text{Old displacement}$$

or

$$\text{New volume} \times \text{new density} = \text{Old volume} \times \text{old density}$$

$$\therefore \frac{\text{New volume}}{\text{Old volume}} = \frac{\text{Old density}}{\text{New density}}$$

With ship-shapes this formula should not be simplified further as it was in the case of a box-shape because the underwater volume is not rectangular. To find the change in draft of a ship-shape due to change of density a quantity known as the 'Fresh Water Allowance' must be known.

The *Fresh Water Allowance* is the number of millimetres by which the mean draft changes when a ship passes from salt water to fresh water, or vice versa, whilst floating at the loaded draft. It is found by the formula:

$$\text{FWA (in mm)} = \frac{\text{Displacement (in tonnes)}}{4 \times \text{TPC}}$$

The proof of this formula is as follows:

$$\text{To show that FWA (in mm)} = \frac{\text{Displacement (in tonnes)}}{4 \times \text{TPC}}$$

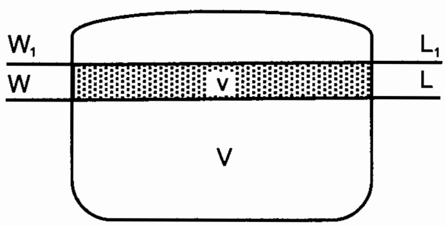

Fig. 5.2

Consider the ship shown in Figure 5.2 to be floating at the load Summer draft in salt water at the waterline WL. Let V be the volume of salt water displaced at this draft.

Now let W_1L_1 be the waterline for the ship when displacing the same mass of fresh water. Also, let 'v' be the extra volume of water displaced in fresh water.

The total volume of fresh water displaced is then V + v.

$$\text{Mass} = \text{Volume} \times \text{density}$$

$$\therefore \text{Mass of SW displaced} = 1025\,V$$

$$\text{and mass of FW displaced} = 1000\,(V + v)$$

$$\text{but mass of FW displaced} = \text{mass of SW displaced}$$

$$\therefore 1000\,(V + v) = 1025\,V$$

$$1000\,V + 1000\,v = 1025\,V$$

$$1000\,v = 25\,V$$

$$v = V/40$$

Now let w be the mass of salt water in volume v, in tonnes and let W be the mass of salt water in volume V, in tonnes.

$$\therefore w = W/40$$

$$\text{but } w = \frac{\text{FWA}}{10} \times \text{TPC}$$

$$\frac{\text{FWA}}{10} \times \text{TPC} = W/40$$

or

$$\text{FWA} = \frac{W}{4 \times \text{TPC}} \text{ mm}$$

where

W = Loaded salt water displacement in tonnes

Figure 5.3 shows a ship's load line marks. The centre of the disc is at a distance below the deck line equal to the ship's Statutory Freeboard. Then 540 mm forward of the disc is a vertical line 25 mm thick, with horizontal lines measuring 230 × 25 mm on each side of it. The upper edge of the one marked 'S' is in line with the horizontal line through the disc and indicates the draft to which the ship may be loaded when floating in salt water in a Summer Zone. Above this line and pointing aft is another line marked 'F', the upper edge of which indicates the draft to which the ship may be loaded when floating in fresh water in a Summer Zone. If loaded to this draft in fresh water the ship will automatically rise to 'S' when she passes into salt water. The perpendicular distance in millimetres between the upper edges of these two lines is therefore the ship's Fresh Water Allowance.

When the ship is loading in dock water which is of a density between these two limits 'S' may be submerged such a distance that she will automatically rise to 'S' when the open sea and salt water is reached. The

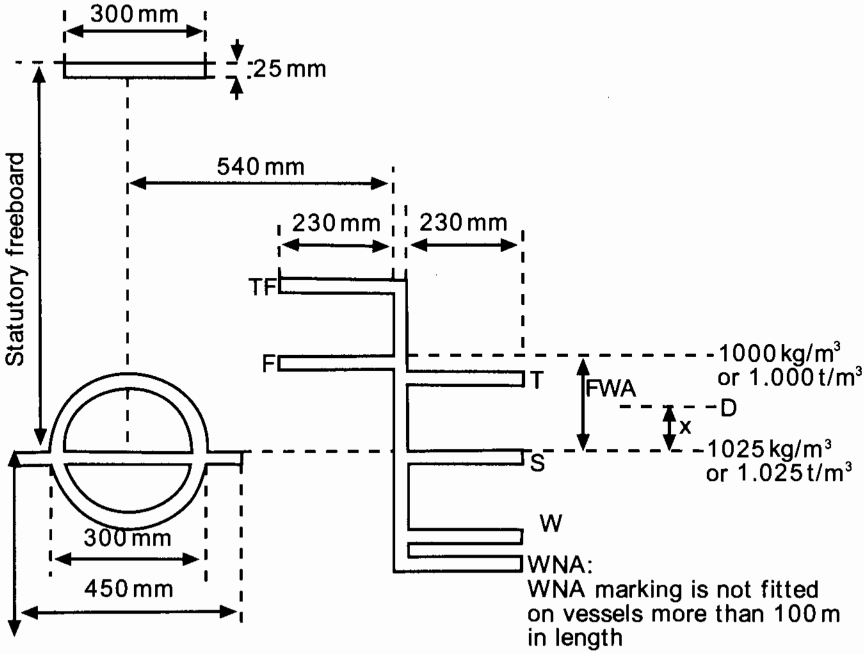

Fig. 5.3

distance by which 'S' can be submerged, called the *Dock Water Allowance*, is found in practice by simple proportion as follows:

Let x = The Dock Water Allowance

Let ρ_{DW} = Density of the dock water

Then

$$\frac{x\,\text{mm}}{\text{FWA}\,\text{mm}} = \frac{1025 - \rho_{DW}}{1025 - 1000}$$

or

$$\text{Dock Water Allowance} = \frac{\text{FWA}\,(1025 - \rho_{DW})}{25}$$

Example 3

A ship is loading in dock water of density 1010 kg per cu. m. FWA = 150 mm. Find the change in draft on entering salt water.

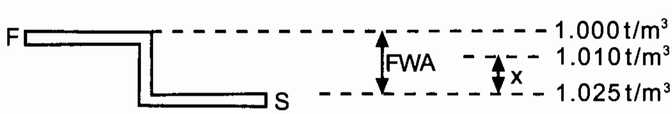

Fig. 5.4

Let x = The change in draft in millimetres

$$\text{Then } \frac{x}{FWA} = \frac{1025 - 1010}{25}$$

$$x = 150 \times \frac{15}{25}$$

$$x = 90\,\text{mm}$$

Ans. Draft will decrease by 90 mm, i.e. 9 cm

Example 4

A ship is loading in a Summer Zone in dock water of density 1005 kg per cu. m. FWA = 62.5 mm, TPC = 15 tonnes. The lower edge of the Summer load line is in the waterline to port and is 5 cm above the waterline to starboard. Find how much more cargo may be loaded if the ship is to be at the correct load draft in salt water.

Note. This ship is obviously listed to port and if brought upright the lower edge of the 'S' load line on each side would be 25 mm above the waterline. Also, it is the upper edge of the line which indicates the 'S' load draft and, since the line is 25 mm thick, the ship's draft must be increased by 50 mm to bring her to the 'S' load line in dock water. In addition 'S' may be submerged by x mm.

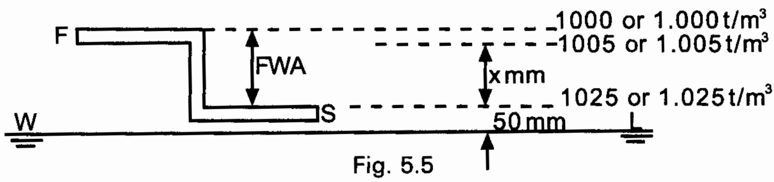

Fig. 5.5

$$\frac{x}{FWA} = \frac{1025 - \rho_{DW}}{25}$$

$$x = 62.5 \times \frac{20}{25}$$

$$x = 50\,\text{mm}$$

∴ Total increase in draft required = 100 mm or 10 cm

and cargo to load = Increase in draft × TPC

= 10 × 15

Ans. Cargo to load = 150 tonnes

Effect of density on displacement when the draft is constant

Should the density of the water in which a ship floats be changed without the ship altering her draft, then the mass of water displaced must have changed. The change in the mass of water displaced may have been brought about by bunkers and stores being loaded or consumed during a sea passage, or by cargo being loaded or discharged.

In all cases:

$$\text{New volume of water displaced} = \text{Old volume of water displaced}$$

or

$$\frac{\text{New displacement}}{\text{New density}} = \frac{\text{Old displacement}}{\text{Old density}}$$

or

$$\frac{\text{New displacement}}{\text{Old displacement}} = \frac{\text{New density}}{\text{Old density}}$$

Example 1

A ship displaces 7000 tonnes whilst floating in fresh water. Find the displacement of the ship when floating at the same draft in water of density 1015 kg per cubic metre, i.e. 1.015 t/m^3.

$$\frac{\text{New displacement}}{\text{Old displacement}} = \frac{\text{New density}}{\text{Old density}}$$

$$\text{New displacement} = \text{Old displacement} \times \frac{\text{New density}}{\text{Old density}}$$

$$= 7000 \times \frac{1015}{1000}$$

Ans. New displacement = 7105 tonnes

Example 2

A ship of 6400 tonnes displacement is floating in salt water. The ship has to proceed to a berth where the density of the water is 1008 kg per cu. m. Find how much cargo must be discharged if she is to remain at the salt water draft.

$$\frac{\text{New displacement}}{\text{Old displacement}} = \frac{\text{New density}}{\text{Old density}}$$

or

$$\text{New displacement} = \text{Old displacement} \times \frac{\text{New density}}{\text{Old density}}$$

$$= 6400 \times \frac{1008}{1025}$$

New displacement = 6293.9 tonnes

Old displacement = 6400.0 tonnes

Ans. Cargo to discharge = 106.1 tonnes

Example 3

A ship 120 m × 17 m × 10 m has a block coefficient 0.800 and is floating at the load Summer draft of 7.2 metres in fresh water. Find how much more cargo can be loaded to remain at the same draft in salt water.

$$\text{Old displacement} = L \times B \times \text{draft} \times C_b \times \text{density}$$

$$= 120 \times 17 \times 7.2 \times 0.800 \times 1000 \text{ tonnes}$$

$$\text{Old displacement} = 11\,750 \text{ tonnes}$$

$$\frac{\text{New displacement}}{\text{Old displacement}} = \frac{\text{New density}}{\text{Old density}}$$

$$\text{New displacement} = \text{Old displacement} \times \frac{\text{New density}}{\text{Old density}}$$

$$= 11,750 \times \frac{1025}{1000}$$

$$\text{New displacement} = 12\,044 \text{ tonnes}$$

$$\text{Old displacement} = 11\,750 \text{ tonnes}$$

Ans. Cargo to load = 294 tonnes

Note. This problem should not be attempted as one involving TPC and FWA.

Exercise 5

Density and draft

1 A ship displaces 7500 cu. m of water of density 1000 kg per cu. m. Find the displacement in tonnes when the ship is floating at the same draft in water of density 1015 kg per cu. m.

2 When floating in fresh water at a draft of 6.5 m a ship displaces 4288 tonnes. Find the displacement when the ship is floating at the same draft in water of density 1015 kg per cu. m.

3 A box-shaped vessel 24 m × 6 m × 3 m displaces 150 tonnes of water. Find the draft when the vessel is floating in salt water.

4 A box-shaped vessel draws 7.5 m in dock water of density 1006 kg per cu. m. Find the draft in salt water of density 1025 kg per cu. m.

5 The KB of a rectangular block which is floating in fresh water is 50 cm. Find the KB in salt water.

6 A ship is lying at the mouth of a river in water of density 1024 kg per cu. m and the displacement is 12 000 tonnes. The ship is to proceed up river and to berth in dock water of density 1008 kg per cu. m with the same draft as at present. Find how much cargo must be discharged.

7 A ship arrives at the mouth of a river in water of density 1016 kg per cu. m with a freeboard of 'S' m. She then discharges 150 tonnes of cargo, and proceeds up river to a second port, consuming 14 tonnes of bunkers. When she arrives at the second port the freeboard is again 'S' m., the density of the water being 1004 kg per cu. m. Find the ship's displacement on arrival at the second port.

8 A ship loads in fresh water to her salt water marks and proceeds along a river to a second port consuming 20 tonnes of bunkers. At the second port, where the density is 1016 kg per cu. m, after 120 tonnes of cargo have been loaded, the ship is again at the load salt water marks. Find the ship's load displacement in salt water.

The TPC and FWA etc.

9 A ship's draft is 6.40 metres forward, and 6.60 metres aft. FWA = 180 mm. Density of the dock water is 1010 kg per cu. m. If the load mean draft in salt water is 6.7 metres, find the final drafts F and A in dock water if this ship is to be loaded down to her marks and trimmed 0.15 metres by the stern. (Centre of flotation is amidships).

10 A ship floating in dock water of density 1005 kg per cu. m has the lower edge of her Summer load line in the waterline to starboard and 50 mm above the waterline to port. FWA = 175 mm and TPC = 12 tonnes. Find the amount of cargo which can yet be loaded in order to bring the ship to the load draft in salt water.

11 A ship is floating at 8 metres mean draft in dock water of relative density 1.01. TPC = 15 tonnes, and FWA = 150 mm. The maximum permissible draft in salt water is 8.1 m. Find the amount of cargo yet to load.

12 A ship's light displacement is 3450 tonnes and she has on board 800 tonnes of bunkers. She loads 7250 tonnes of cargo, 250 tonnes of bunkers, and 125 tonnes of fresh water. The ship is then found to be 75 mm from the load draft. TPC = 12 tonnes. Find the ship's deadweight and load displacement.

13 A ship has a load displacement of 5400 tonnes, TPC = 30 tonnes. If she loads to the Summer load line in dock water of density 1010 kg per cu. m, find the change in draft on entering salt water of density 1025 kg per cu. m.

14 A ship's FWA is 160 mm, and she is floating in dock water of density 1012 kg per cu. m. Find the change in draft when she passes from dock water to salt water.

Chapter 6
Transverse statical stability

Recapitulation

1. The centre of gravity of a body 'G' is the point through which the force of gravity is considered to act vertically downwards with a force equal to the weight of the body. KG is VCG of the ship.
2. The centre of buoyancy 'B' is the point through which the force of buoyancy is considered to act vertically upwards with a force equal to the weight of water displaced. It is the centre of gravity of the underwater volume. KB is VCB of the ship.
3. To float at rest in still water, a vessel must displace her own weight of water, and the centre of gravity must be in the same vertical line as the centre of buoyancy.
4. KM = KB + BM. Also KM = KG + GM.

Definitions

1. *Heel.* A ship is said to be heeled when she is inclined by an external force. For example, when the ship is inclined by the action of the waves or wind.
2. *List.* A ship is said to be listed when she is inclined by forces within the ship. For example, when the ship is inclined by shifting a weight transversely within the ship. This is a fixed angle of heel.

The metacentre

Consider a ship floating upright in still water as shown in Figure 6.1(a). The centres of gravity and buoyancy are at G and B respectively. Figure 6.1(c) shows the righting couple. GZ is the righting lever.

Now let the ship be inclined by an external force to a small angle (θ) as shown in Figure 6.1(b). Since there has been no change in the distribution of weights the centre of gravity will remain at G and the weight of the ship (W) can be considered to act vertically downwards through this point.

When heeled, the wedge of buoyancy WOW_1 is brought out of the water and an equal wedge LOL_1 becomes immersed. In this way a wedge of buoyancy having its centre of gravity at g is transferred to a position with its

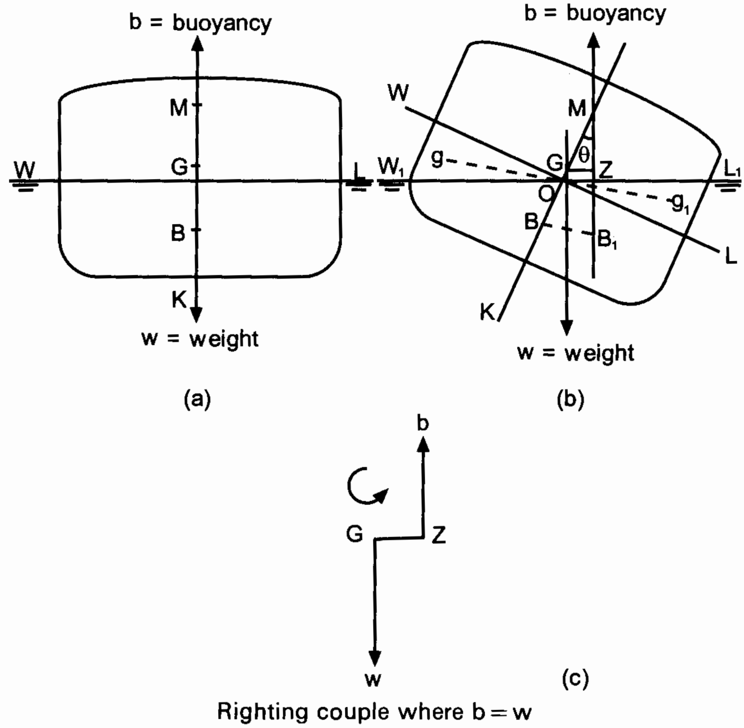

Fig. 6.1. Stable equilibrium

centre of gravity at g_1. The centre of buoyancy, being the centre of gravity of the underwater volume, must shift from B to the new position B_1, such that BB_1 is parallel to gg_1, and $BB_1 = \frac{v \times gg_1}{V}$ where v is the volume of the transferred wedge, and V is the ship's volume of displacement.

The verticals through the centres of buoyancy at two consecutive angles of heel intersect at a point called the *metacentre*. For angles of heel up to about 15° the vertical through the centre of buoyancy may be considered to cut the centre line at a fixed point called the initial metacentre (M in Figure 6.1(b)). The height of the initial metacentre above the keel (KM) depends upon a ship's underwater form. Figure 6.2 shows a typical curve of KM's for a ship plotted against draft.

The vertical distance between G and M is referred to as the *metacentric height*. If G is below M the ship is said to have positive metacentric height, and if G is above M the metacentric height is said to be negative.

Equilibrium

Stable equilibrium

A ship is said to be in stable equilibrium if, when inclined, she tends to return to the initial position. For this to occur the centre of gravity must be

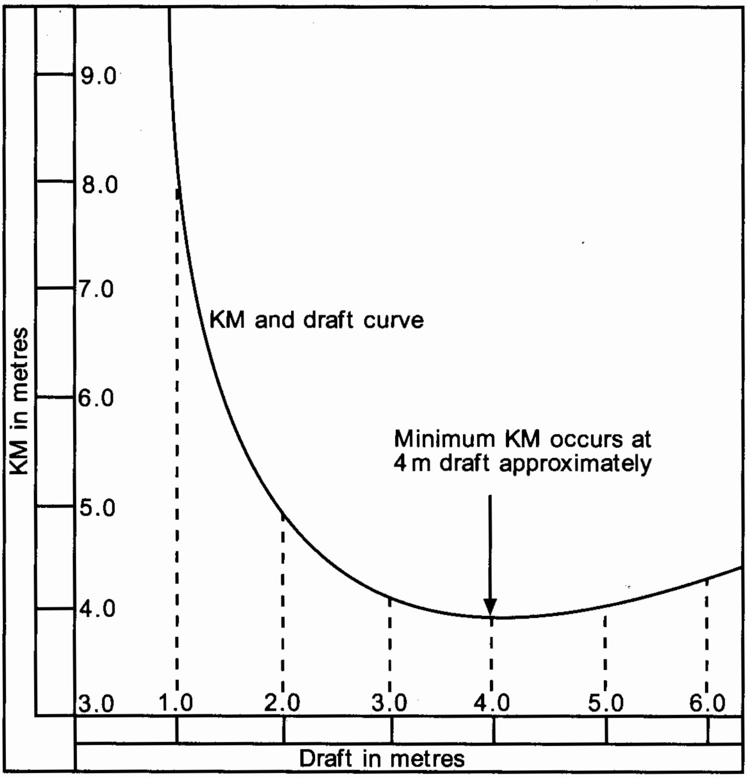

Fig. 6.2

below the metacentre, that is, the ship must have positive initial metacentric height. Figure 6.1(a) shows a ship in the upright position having a positive GM. Figure 6.1(b) shows the same ship inclined to a small angle. The position of G remains unaffected by the heel and the force of gravity is considered to act vertically downwards through this point. The centre of buoyancy moves out to the low side from B to B_1 to take up the new centre of gravity of the underwater volume, and the force of buoyancy is considered to act vertically upwards through B_1 and the metacentre M. If moments are taken about G there is a moment to return the ship to the upright. This moment is referred to as the *Moment of Statical Stability* and is equal to the product of the force 'W' and the length of the lever GZ. i.e.

Moment of Statical Stability $= W \times GZ$ *tonnes-metres.*

The lever GZ is referred to as the *righting lever* and is the perpendicular distance between the centre of gravity and the vertical through the centre of buoyancy.

At a small angle of heel (less than 15°):

$GZ = GM \times \sin\theta$ and Moment of Statical Stability $= W \times GM \times \sin\theta$

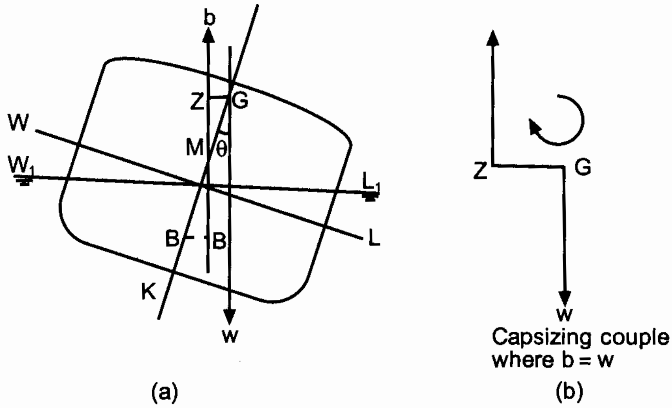

Fig. 6.3. Unstable equilibrium.

Unstable equilibrium

When a ship which is inclined to a small angle tends to heel over still further, she is said to be in unstable equilibrium. For this to occur the ship must have a negative GM. Note how G is above M.

Figure 6.3(a) shows a ship in unstable equilibrium which has been inclined to a small angle. The moment of statical stability, W × GZ, is clearly a capsizing moment which will tend to heel the ship still further.

Note. A ship having a very small negative initial metacentric height GM need not necessarily capsize. This point will be examined and explained later. This situation produces an angle of loll.

Neutral equilibrium

When G coincides with M as shown in Figure 6.4(a), the ship is said to be in neutral equilibrium, and if inclined to a small angle she will tend to remain

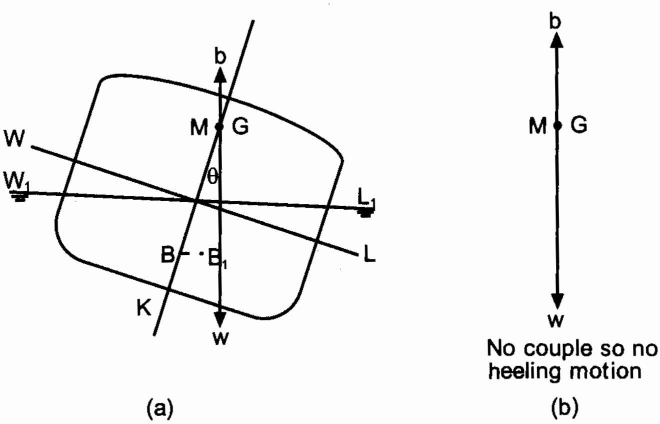

Fig. 6.4. Neutral equilibrium.

at that angle of heel until another external force is applied. The ship has zero GM. Note that KG = KM.

$$\text{Moment of Statical Stability} = W \times GZ, \text{ but in this case } GZ = 0$$

$$\therefore \text{Moment of Statical Stability} = 0 \text{ see Figure 6.4(b)}$$

Therefore there is no moment to bring the ship back to the upright or to heel her over still further. The ship will move vertically up and down in the water at the fixed angle of heel until further external or internal forces are applied.

Correcting unstable and neutral equilibrium

When a ship in unstable or neutral equilibrium is to be made stable, thc effective centre of gravity of the ship should be lowered. To do this one or more of the following methods may be employed:

1. weights already in the ship may be lowered,
2. weights may be loaded below the centre of gravity of the ship,
3. weights may be discharged from positions above the centre of gravity, or
4. free surfaces within the ship may be removed.

The explanation of this last method will be found in Chapter 7.

Stiff and tender ships

The *time period* of a ship is the time taken by the ship to roll from one side to the other and back again to the initial position.

When a ship has a comparatively large GM, for example 2 m to 3 m, the righting moments at small angles of heel will also be comparatively large. It will thus require larger moments to incline the ship. When inclined she will tend to return more quickly to the initial position. The result is that the ship will have a comparatively short time period, and will roll quickly – and perhaps violently – from side to side. A ship in this condition is said to be 'stiff', and such a condition is not desirable. The time period could be as low as 8 seconds. The effective centre of gravity of the ship should be raised within that ship.

When the GM is comparatively small, for example 0.16 m to 0.20 m the righting moments at small angles of heel will also be small. The ship will thus be much easier to incline and will not tend to return so quickly to the initial position. The time period will be comparatively long and a ship, for example 30 to 35 seconds, in this condition is said to be 'tender'. As before, this condition is not desirable and steps should be taken to increase the GM by lowering the effective centre of gravity of the ship.

The officer responsible for loading a ship should aim at a happy medium between these two conditions whereby the ship is neither too stiff nor too tender. A time period of 20 to 25 seconds would generally be acceptable for those on board a ship at sea.

Negative GM and angle of loll

It has been shown previously that a ship having a negative initial metacentric height will be unstable when inclined to a small angle. This is shown in Figure 6.5(a).

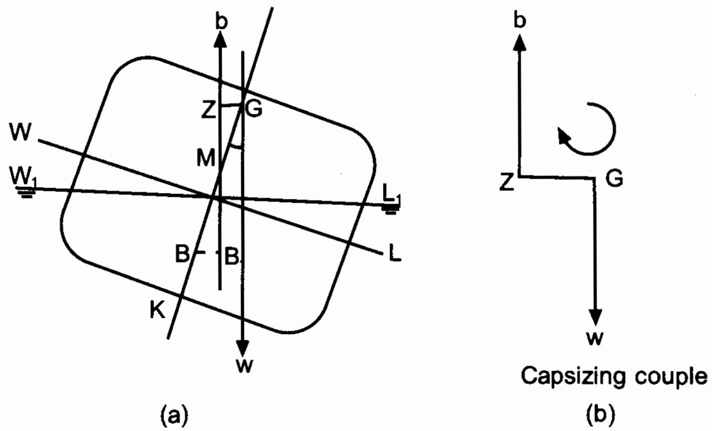

Fig. 6.5

As the angle of heel increases, the centre of buoyancy will move out still further to the low side. If the centre of buoyancy moves out to a position vertically under G, the capsizing moment will have disappeared as shown in Figure 6.5(b). The angle of heel at which this occurs is called the *angle of loll.* It will be noticed that at the angle of loll, the GZ is zero. G remains on the centre line.

If the ship is heeled beyond the angle of loll from θ_1 to θ_2, the centre of buoyancy will move out still further to the low side and there will be a moment to return her to the angle of loll as shown in Figure 6.5(c).

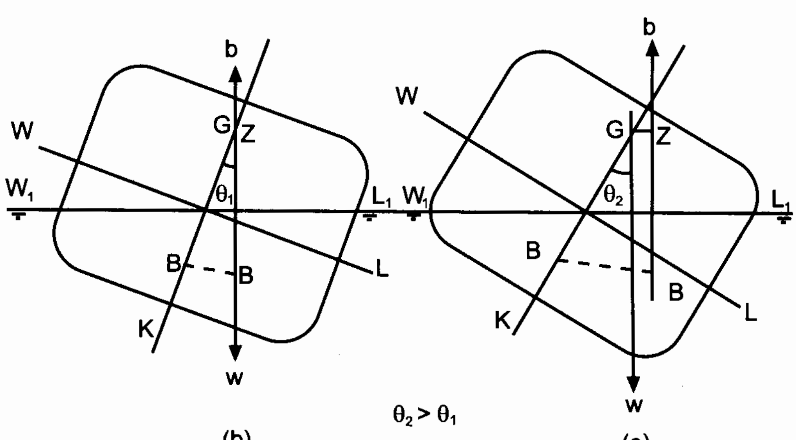

Fig. 6.5

From this it can be seen that the ship will oscillate about the angle of loll instead of about the vertical. If the centre of buoyancy does not move out far enough to get vertically under G, the ship will capsize.

The angle of loll will be to port or starboard and back to port depending on external forces such as wind and waves. One minute it may flop over to 3° P and then suddenly flop over to 3° S.

There is always the danger that G will rise above M and create a situation of unstable equilibrium. This will cause capsizing of the ship.

Exercise 6

1 Define the terms 'heel', 'list', 'initial metacentre' and 'initial metacentric height'.
2 Sketch transverse sections through a ship, showing the positions of the centre of gravity, centre of buoyancy, and initial metacentre, when the ship is in (a) Stable equilibrium, (b) Unstable equilibrium, and (c) Neutral equilibrium.
3 Explain what is meant by a ship being (a) tender and, (b) stiff;
4 With the aid of suitable sketches, explain what is meant by 'angle of loll'.
5 A ship of 10 000 t displacement has an initial metacentric height of 1.5 m. What is the moment of statical stability when the ship is heeled 10 degrees?

The GM value

GM is crucial to ship stability. The table below shows *typical* working values for GM for several ship-types all at *fully-loaded* drafts.

Ship type	GM at fully-loaded condition
General cargo ships	0.30–0.50 m
Oil tankers to VLCCs	0.30–1.00 m
Container ships	1.50 m approx.
Ro-Ro vessels	1.50 m approx.
Bulk ore carriers	2–3 m

At drafts below the fully-loaded draft, due to KM tending to be larger in value it will be found that corresponding GM values will be *higher* than those listed in the table above. For all conditions of loading the D.Tp stipulate that the GM must never be less than 0.15 m.