Chapter 28
Drydocking and grounding

When a ship enters a drydock she must have a positive initial GM, be upright, and trimmed slightly, usually by the stern. On entering the drydock the ship is lined up with her centre line vertically over the centre line of the keel blocks and the shores are placed loosely in position. The dock gates are then closed and pumping out commences. The rate of pumping is reduced as the ship's stern post nears the blocks. When the stern lands on the blocks the shores are hardened up commencing from aft and gradually working forward so that all of the shores will be hardened up in position by the time the ship takes the blocks overall. The rate of pumping is then increased to quickly empty the dock.

As the water level falls in the drydock there is no effect on the ship's stability so long as the ship is completely waterborne, but after the stern lands on the blocks the draft aft will decrease and the trim will change by the head. This will continue until the ship takes the blocks overall throughout her length, when the draft will then decrease uniformly forward and aft.

The interval of time between the stern post landing on the blocks and the ship taking the blocks overall is referred to as the *critical period*. During this period part of the weight of the ship is being borne by the blocks, and this creates an upthrust at the stern which increases as the water level falls in the drydock. The upthrust causes a virtual loss in metacentric height and it is essential that positive effective metacentric height be maintained throughout the critical period, or the ship will heel over and perhaps slip off the blocks with disastrous results.

The purpose of this chapter is to show the methods by which the effective metacentric height may be calculated for any instant during the drydocking process.

Figure 28.1 shows the longitudinal section of a ship during the critical period. 'P' is the upthrust at the stern and 'l' is the distance of the centre of flotation from aft. The trimming moment is given by $P \times l$. But the trimming moment is also equal to MCTC × Change of trim.

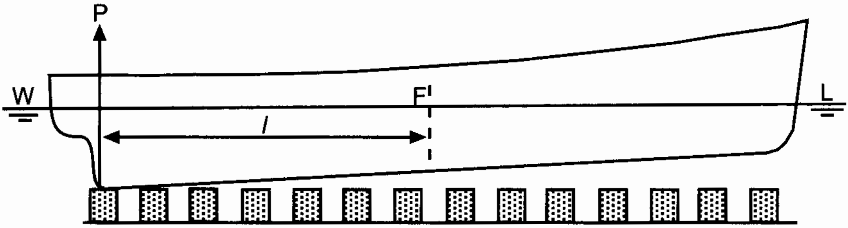

Fig. 28.1

Therefore,

$$P \times l = MCTC \times t$$

or,

$$P = \frac{MCTC \times t}{l}$$

where

P = the upthrust at the stern in tonnes,

t = the change of trim since entering the drydock in centimetres, and

l = the distance of the centre of flotation from aft in metres.

Now consider Figure 28.2 which shows a transverse section of the ship during the critical period after she has been inclined to a small angle (θ degrees) by a force external to the ship. For the sake of clarity the angle of heel has been magnified. The weight of the ship (W) acts downwards through the centre of gravity (G). The force P acts upwards through the keel (K) and is equal to the weight being borne by the blocks. For equilibrium the force of buoyancy must now be (W − P) and will act upwards through the initial metacentre (M).

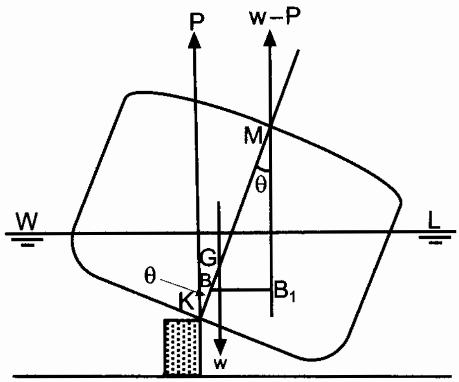

Fig. 28.2

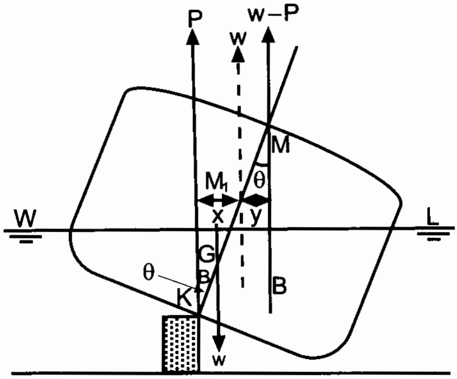

Fig. 28.3

There are, thus, three parallel forces to consider when calculating the effect of the force P on the ship's stability. Two of these forces may be replaced by their resultant (see page 4) in order to find the effective metacentric height and the moment of statical stability.

Method (a)

In Figure 28.3 consider the two parallel forces P and (W − P). Their resultant W will act upwards through M_1 such that:

$$(W - P) \times y = P \times X$$

or

$$(W - P) \times MM_1 \times \sin\theta = P \times KM_1 \times \sin\theta$$

$$(W - P) \times MM_1 = P \times KM_1$$

$$W \times MM_1 - P \times MM_1 = P \times KM_1$$

$$W \times MM_1 = P \times KM_1 + P \times MM_1$$

$$= P\,(KM_1 + MM_1)$$

$$= P \times KM$$

$$MM_1 = \frac{P \times KM}{W}$$

There are now two forces to consider: W acting upwards through M_1 and W acting downwards through G. These produce a righting moment of $W \times GM_1 \times \sin\theta$.

Note also that the original metacentric height was GM but has now been reduced to GM_1. Therefore MM_1 is the virtual loss of metacentric height due to drydocking.

Or

$$\textit{Virtual loss of GM } (MM_1) = \frac{P \times KM}{W}$$

Method (b)

Now consider the two parallel forces W and P in Figure 28.4. Their resultant (W − P) acts downwards through G_1 such that:

$$W \times y = P \times X$$

or

$$W \times GG_1 \times \sin\theta = P \times KG_1 \times \sin\theta$$

$$W \times GG_1 = P \times KG_1$$

$$= P(KG + GG_1)$$

$$= P \times KG + P \times GG_1$$

$$W \times GG_1 - P \times GG_1 = P \times KG$$

$$GG_1 (W - P) = P \times KG$$

$$GG_1 = \frac{P \times KG}{W - P}$$

There are now two forces to consider: (W − P) acting upwards through M, and (W − P) acting downwards through G_1. These produce a righting moment of $(W - P) \times G_1M \times \sin\theta$.

The original metacentric height was GM but has now been reduced to G_1M. Therefore GG_1 is the virtual loss of metacentric height due to drydocking.

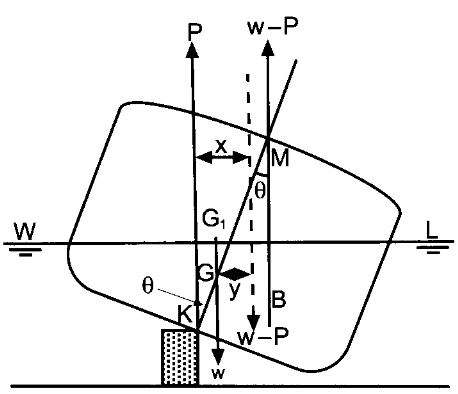

Fig. 28.4

Or

$$\textit{Virtual loss of GM } (GG_1) = \frac{P \times KG}{W - P}$$

Example 1

A ship of 6000 tonnes displacement enters a drydock trimmed 0.3 m by the stern. KM = 7.5 m, KG = 6 m. MCTC = 90 tonnes m. The centre of flotation is 45 m from aft. Find the effective metacentric height at the critical instant before the ship takes the blocks overall.

Note. Assume that the trim at the critical instant is zero.

$$P = \frac{MCTC \times t}{l}$$

$$= \frac{90 \times 30}{45}$$

$$P = 60 \text{ tonnes}$$

Method (a)

$$\text{Virtual loss of GM } (MM_1) = \frac{P \times KM}{W}$$

$$= \frac{60 \times 7.5}{6000}$$

$$= 0.075 \text{ m}$$

$$\text{Original GM} = 7.5 - 6.0 = 1.500 \text{ m}$$

Ans. New GM = 1.425 m

Method (b)

$$\text{Virtual loss of GM} = \frac{P \times KG}{W - P}$$

$$GG_1 = \frac{60 \times 6}{5940}$$

$$= 0.061 \text{ m}$$

$$\text{Original GM} = 1.500 \text{ m}$$

Ans. New GM = 1.439 m

From these results it would appear that there are two possible answers to the same problem, but this is not the case. The ship's ability to return to the upright is indicated by the righting moment and not by the effective metacentric height alone.

To illustrate this point, calculate the righting moments given by each method when the ship is heeled to a small angle (θ°)

Method (a)

$$\text{Righting moment} = W \times GM_1 \times \sin\theta$$

$$= 6000 \times 1.425 \times \sin\theta^\circ$$

$$= (8550 \times \sin\theta^\circ) \text{ tonnes m.}$$

Method (b)

$$\text{Righting moment} = (W - P) \times G_1M \times \sin\theta$$
$$= 5940 \times 1.439 \times \sin\theta^\circ$$
$$= (8549 \times \sin\theta) \text{ tonnes metres.}$$

Thus each of the two methods used gives a correct indication of the ship's stability during the critical period.

Example 2

A ship of 3000 tonnes displacement is 100 m long, has KM = 6 m, KG = 5.5 m. The centre of flotation is 2 m aft of amidships. MCTC = 40 tonnes m. Find the maximum trim for the ship to enter a drydock if the metacentric height at the critical instant before the ship takes the blocks forward and aft is to be not less than 0.3 m.

$$KM = 6.0\ \text{m}$$
$$KG = 5.5\ \text{m}$$
$$\text{Original GM} = 0.5\ \text{m}$$
$$\text{Virtual GM} = 0.3\ \text{m}$$
$$\underline{\text{Virtual loss} = 0.2\ \text{m}}$$

Method (a)

$$\text{Virtual loss of GM } (MM_1) = \frac{P \times KM}{W}$$

or

$$P = \frac{\text{Virtual loss} \times W}{KM}$$
$$= \frac{0.2 \times 3000}{6}$$
$$\underline{\text{Maximum } P = 100 \text{ tonnes}}$$

But

$$P = \frac{MCTC \times t}{l}$$

or

$$\text{Maximum } t = \frac{P \times l}{MCTC}$$
$$= \frac{100 \times 48}{40}$$

Ans. $\underline{\text{Maximum trim} = 120\ \text{cm by the stern}}$

Method (b)

$$\text{Virtual loss of GM (GG}_1) = \frac{P \times KG}{W - P}$$

$$0.2 = \frac{P \times 5.5}{3000 - P}$$

$$600 - 0.2\,P = 5.5\,P$$

$$5.7\,P = 600$$

$$\text{Maximum P} = \frac{600}{5.7} = 105.26 \text{ tonnes}$$

But

$$P = \frac{MCTC \times t}{l}$$

or

$$\text{Maximum t} = \frac{P \times l}{MCTC}$$

$$= \frac{105.26 \times 48}{40}$$

Ans. Maximum trim = 126.3 cm by the stern

There are therefore two possible answers to this question, depending upon the method of solution used. The reason for this is that although the effective metacentric height at the critical instant in each case will be the same, the righting moments at equal angles of heel will not be the same.

Example 3

A ship of 5000 tonnes displacement enters a drydock trimmed 0.45 m by the stern. KM = 7.5 m, KG = 6.0 m. MCTC = 120 tonnes m. The centre of flotation is 60 m from aft. Find the effective metacentric height at the critical instant before the ship takes the blocks overall, assuming that the transverse metacentre rises 0.075 m.

$$P = \frac{MCTC \times t}{l}$$

$$= \frac{120 \times 45}{60}$$

$$P = 90 \text{ tonnes}$$

Method (a)

$$\text{Virtual loss (MM}_1) = \frac{P \times KM}{W}$$

$$= \frac{90 \times 7.575}{5000}$$

$$= 0.136\text{ m}$$

$$\text{Original KM} = 7.500\,\text{m}$$
$$\text{Rise of M} = 0.075\,\text{m}$$
$$\text{New KM} = 7.575\,\text{m}$$
$$\text{KG} = 6.000\,\text{m}$$
$$\text{GM} = 1.575\,\text{m}$$
$$\text{Virtual loss } (MM_1) = 0.136\,\text{m}$$

Ans. New GM = 1.439 m

Method (b)

$$\text{Virtual loss } (GG_1) = \frac{P \times KG}{W - P}$$
$$= \frac{90 \times 6.0}{4910}$$
$$= 0.110\,\text{m}$$
$$\text{Old KG} = 6.000\,\text{m}$$
$$\text{Virtual loss } (GG_1) = 0.110\,\text{m}$$
$$\text{New KG} = 6.110\,\text{m}$$
$$\text{New KM} = 7.575\,\text{m}$$

Ans. New GM = 1.465 m

The virtual loss of GM after taking the blocks overall

When a ship takes the blocks overall, the water level will then fall uniformly about the ship, and for each centimetre fallen by the water level P will be increased by a number of tonnes equal to the TPC. Also, the force P at any time during the operation will be equal to the difference between the weight of the ship and the weight of water she is displacing at that time.

Example 4

A ship of 5000 tonnes displacement enters a drydock on an even keel. KM = 6 m. KG = 5.5 m, and TPC = 50 tonnes. Find the virtual loss of metacentric height after the ship has taken the blocks and the water has fallen another 0.24 m.

$$P = \text{TPC} \times \text{reduction in draft in cm}$$
$$= 50 \times 24$$
$$P = 1200\ \text{tonnes}$$

Method (a)

$$\text{Virtual loss}(MM_1) = \frac{P \times KM}{W}$$

$$= \frac{1200 \times 6}{5000}$$

Ans. Virtual loss = 1.44 m

Method (b)

$$\text{Virtual loss } (GG_1) = \frac{P \times KG}{W - P}$$

$$= \frac{1200 \times 5.5}{3800}$$

Ans. Virtual loss = 1.74 m

Note to Students

In the D.Tp. examinations, when sufficient information is given in a question, either method of solution may be used. It has been shown in this chapter that both are equally correct. In some questions, however, there is no choice, as the information given is sufficient for only one of the methods to be used. It is therefore advisable for students to learn both of the methods.

Example 5

A ship of 8000 tonnes displacement takes the ground on a sand bank on a falling tide at an even keel draft of 5.2 metres. KG 4.0 metres. The predicted depth of water over the sand bank at the following low water is 3.2 metres. Calculate the GM at this time assuming that the KM will then be 5.0 metres and that the mean TPC is 15 tonnes.

$$P = \text{TPC} \times \text{Fall in water level (cm)} = 15 \times (520 - 320)$$

$$= 15 \times 200$$

$$P = 3000 \text{ tonnes}$$

Method (a)

$$\text{Virtual loss of GM } (MM_1) = \frac{P \times KM}{W}$$

$$= \frac{3000 \times 5}{8000}$$

$$= 1.88 \text{ m}$$

$$\text{Actual KM} = 5.00 \text{ m}$$

$$\text{Virtual KM} = 3.12 \text{ m}$$

$$\text{KG} = 4.00 \text{ m}$$

Ans. New GM = −0.88 m

Method (b)

$$\text{Virtual loss of GM }(GG_1) = \frac{P \times KG}{W - P}$$

$$= \frac{3000 \times 4}{5000}$$

$$= 2.40\,\text{m}$$

$$KG = 4.00\,\text{m}$$

$$\text{Virtual KG} = 6.40\,\text{m}$$

$$KM = 5.00\,\text{m}$$

Ans. New GM = −1.40 m

Note that in Example 5, this vessel has developed a negative GM. Consequently she is *unstable*. She would capsize if transverse external forces such as wind or waves were to remove her from zero angle of heel. Suggest a change of loading to reduce KG and make GM a positive value greater that D.Tp. minimum of 0.15 m.

EXERCISE 28

1 A ship being drydocked has a displacement of 1500 tonnes. TPC = 5 tonnes, KM = 3.5 m, GM = 0.5 m, and has taken the blocks fore and aft at 3 m draft. Find the GM when the water level has fallen another 0.6 m.

2 A ship of 4200 tonnes displacement has GM 0.75 m and present drafts 2.7 m F and 3.7 m A. She is to enter a drydock. MCTC = 120 tonnes m. The after keel block is 60 m aft of the centre of flotation. At 3.2 m mean draft KM = 8 m. Find the GM on taking the blocks forward and aft.

3 A box-shaped vessel 150 m long, 10 m beam, and 5 m deep, has a mean draft in salt water of 3 m and is trimmed 1 m by the stern, KG = 3.5 m. State whether it is safe to drydock this vessel in this condition or not, and give reasons for your answer.

4 A ship of 6000 tonnes displacement is 120 m long and is trimmed 1 m by the stern. KG = 5.3 m, GM = 0,7 m. MCTC = 90 tonnes m. Is it safe to drydock the ship in this condition? (Assume that the centre of flotation is amidships.)

5 A ship of 4000 tonnes displacement, 126 m long, has KM = 6.7 m. KG = 6.1 m. The centre of flotation is 3 m aft of amidships. MCTC = 120 tonnes m. Find the maximum trim at which the ship may enter a drydock if the minimum GM at the critical instant is to be 0.3 m.

Chapter 29
Second moments of areas

The second moment of an element of an area about an axis is equal to the product of the area and the square of its distance from the axis. Let dA in Figure 29.1 represent an element of an area and let y be its distance from the axis AB.

The second moment of the element about AB is equal to $dA \times y^2$.

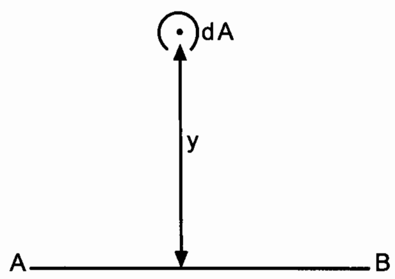

Fig. 29.1

To find the second moment of a rectangle about an axis parallel to one of its sides and passing through the centroid.

In Figure 29.2, *l* represents the length of the rectangle and b represents the breadth. Let G be the centroid and let AB, an axis parallel to one of the sides, pass through the centroid.

Consider the elementary strip which is shown shaded in the figure. The second moment (i) of the strip about the axis AB is given by the equation:

$$i = l \cdot dx \times x^2$$

Let I_{AB} be the second moment of the whole rectangle about the axis AB then:

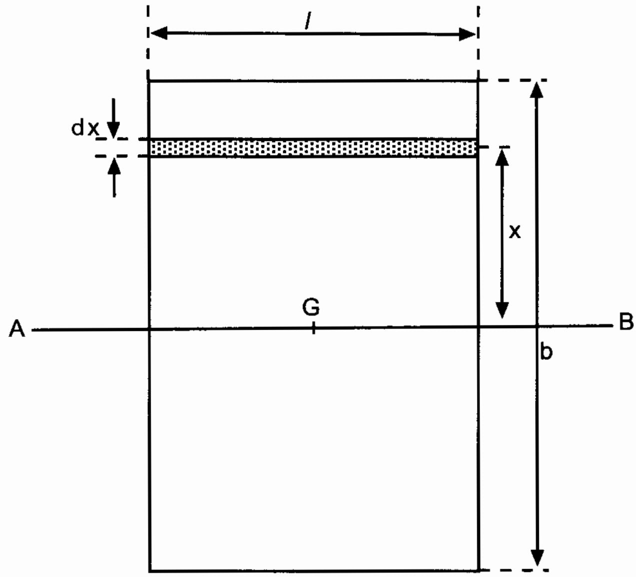

Fig. 29.2

$$I_{AB} = \int_{-b/2}^{+b/2} lx^2\,dx$$

$$I_{AB} = l\int_{-b/2}^{+b/2} x^2\,dx$$

$$= l\left[\frac{x^3}{3}\right]_{-b/2}^{+b/2}$$

$$I_{AB} = \frac{lb^3}{12}$$

To find the second moment of a rectangle about one of its sides.

Consider the second moment (i) of the elementary strip shown in Figure 29.3 about the axis AB.

$$i = l \cdot dx \times x^2$$

Let I_{AB} be the second moment of the rectangle about the axis AB, then:

$$I_{AB} = \int_O^b lx^2\,dx$$

$$= l\left[\frac{x^3}{3}\right]_O^b$$

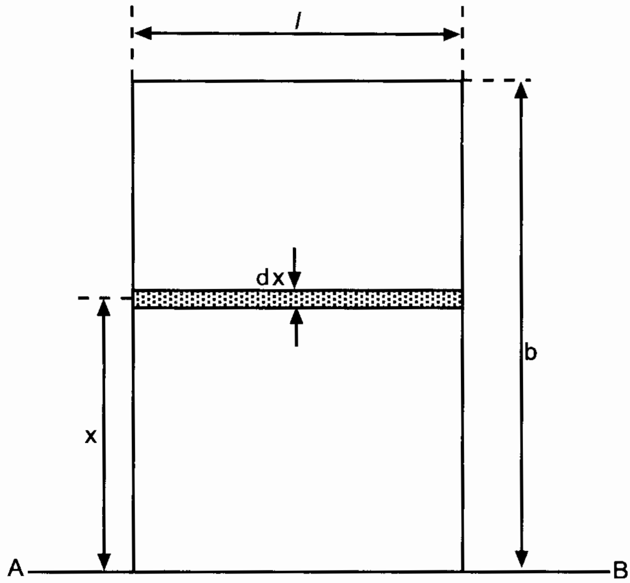

Fig. 29.3

or

$$I_{AB} = \frac{lb^3}{3}$$

The Theorem of Parallel Axes

The second moment of an area about an axis through the centroid is equal to the second moment about any other axis parallel to the first reduced by the product of the area and the square of the perpendicular distance between the two axes. Thus, in Figure 29.4, if G represents the centroid of the area (A) and the axis OZ is parallel to AB, then:

$$I_{OZ} = I_{AB} - Ay^2 = \text{parallel axis theorem equation}$$

To find the second moment of a ship's waterplane area about the centre line.

In Figure 29.5:

$$\text{Area of elementary strip} = y \cdot dx$$

$$\text{Area of waterplane} = \int_O^L y \cdot dx$$

It has been shown in chapter 10 that the area under the curve can be found by Simpson's Rules, using the values of y, the half-breadths, as ordinates.

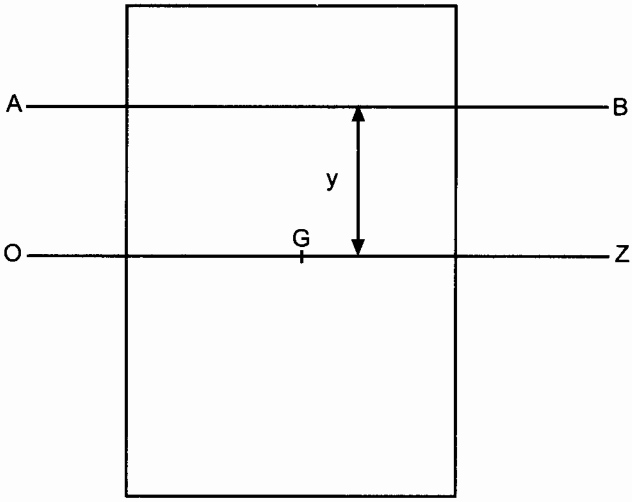

Fig. 29.4

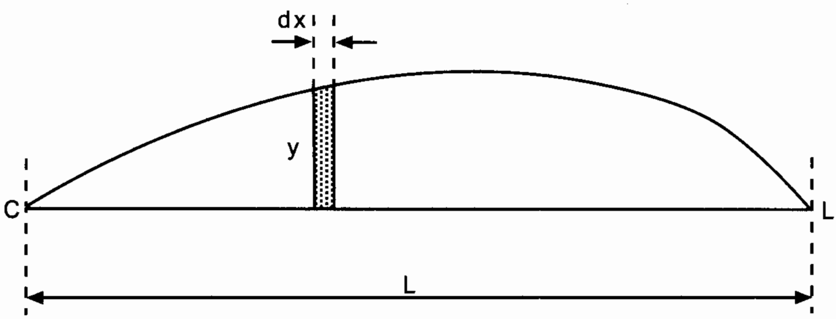

Fig. 29.5

The second moment of a rectangle about one end is given by $\frac{lb^3}{3}$, and therefore the second moment of the elementary strip about the centre line is given by $\frac{y^3\,dx}{3}$ and the second moment of the half waterplane about the centre line is given by

$$\int_O^L \frac{y^3}{3}\,dx$$

Therefore, if I_{CL} is the second moment of the whole waterplane area about the centre line, then:

$$I_{CL} = \frac{2}{3}\int_O^L y^3\,dx$$

The integral part of this expression can be evaluated by Simpson's Rules using the values of y^3 (i.e. the half-breadths cubed), as ordinates, and I_{CL} is found by multiplying the result by $\frac{2}{3}$.

Example 1

A ship's waterplane is 18 metres long. The half-ordinates at equal distances from forward are as follows:

0, 1.2, 1.5, 1.8, 1.8, 1.5, and 1.2 metres,

respectively. Find the second moment of the waterplane area about the centre line.

$\frac{1}{2}$ *ord.*	$\frac{1}{2}$ *ord.*^{3}	*S.M.*	*Products for* I_{CL}
0	0	1	0
1.2	1.728	4	6.912
1.5	3.375	2	6.750
1.8	5.832	4	23.328
1.8	5.832	2	11.664
1.5	3.375	4	13.500
1.2	1.728	1	1.728
			$63.882 = \Sigma_1$

$$I_{CL} = \frac{2}{9} \times CI \times \Sigma_1$$

$$I_{CL} = \frac{2}{9} \times \frac{18}{6} \times 63.882$$

$$= \underline{42.588 \text{ m}^4}$$

To find the second moment of the waterplane area about a transverse axis through the centre of flotation.

$$\text{Area of elementary strip} = y\,dx$$

$$I_{AB} \text{ of the elementary strip} = x^2\,y\,dx$$

$$I_{AB} \text{ of the waterplane area} = 2\int_O^L x^2\,y\,dx$$

Once again the integral part of this expression can be evaluated by Simpson's Rules using the values of $x^2 y$ as ordinates and the second moment about AB is found by multiplying the result by two.

Let OZ be a transverse axis through the centre of flotation. The second moment about OZ can then be found by the theorem of parallel axes. i.e.

$$I_{OZ} = I_{AB} - A\bar{X}^2$$

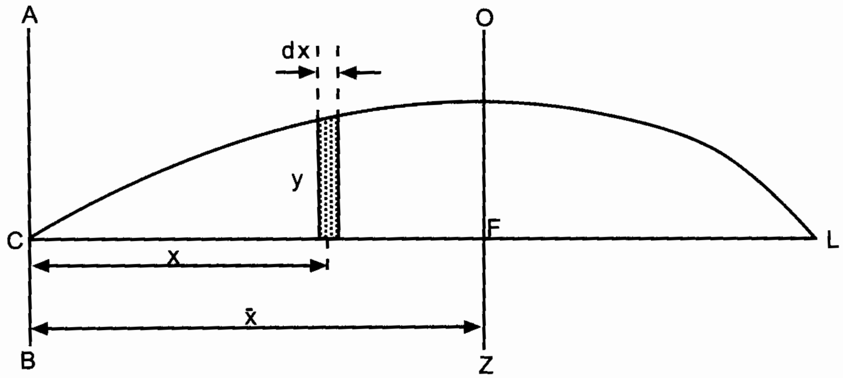

Fig. 29.6

Example 2

A ship's waterplane is 18 metres long. The half-ordinates at equal distances from forward are as follows:

0, 1.2, 1.5, 1.8, 1.8, 1.5, 1.2 metres, respectively.

Find the second moment of the waterplane area about a transverse axis through the centre of flotation.

$\frac{1}{2}$ *ord.*	*SM*	*Area func*	*Lever*	*Moment func*	*Lever*	*Inertia func.*
0	1	0	0	0	0	0
1.2	4	4.8	1	4.8	1	4.8
1.5	2	3.0	2	6.0	2	12.0
1.8	4	7.2	3	21.6	3	64.8
1.8	2	3.6	4	14.4	4	57.6
1.5	4	6.0	5	30.0	5	150.0
1.2	1	1.2	6	7.2	6	43.2
		$25.8 = \Sigma_1$		$84.0 = \Sigma_2$		$332.4 = \Sigma_3$

$$\text{Area of waterplane} = \frac{1}{3} \times \text{CI} \times \Sigma_1 \times 2$$

$$\text{Area of waterplane} = \frac{1}{3} \times \frac{18}{6} \times 25.8 \times 2$$

$$= 51.6\,\text{sq. m}$$

$$\text{Distance of the Centre of Flotation from forward} = \frac{\Sigma_2}{\Sigma_1} \times \text{CI}$$

$$= \frac{84}{25.8} \times \frac{18}{6}$$

$$= 9.77\,\text{m}$$

$$= 0.77\,\text{m aft of amidships}$$

$$I_{AB} = \frac{1}{3} \times (CI)^3 \times \Sigma_3 \times 2$$

$$= \frac{1}{3} \times \left(\frac{18}{6}\right)^3 \times 332.4 \times 2 = 5983\,\text{m}^4$$

$$I_{OZ} = I_{AB} - A\bar{X}^2$$

$$= 5983 - 51.6 \times 9.77^2$$

$$= 5983 - 4925$$

Ans. I_{OZ} = 1058 metres^4

There is a quicker and more efficient method of obtaining the solution to the above problem. Instead of using the foremost ordinate at the datum, use the midship ordinate. Proceed as follows:

$\frac{1}{2}$ *ord.*	*SM*	*Area func.*	*Lever* ⚓	*Moment func.*	*Lever* ⚓	*Inertia func.*
0	1	0	−3	0	−3	0
1.2	4	4.8	−2	−9.6	−2	+19.2
1.5	2	3.0	−1	−3.0	−1	+3.0
1.8	4	7.2	0	0	0	0
1.8	2	3.6	+1	+3.6	+1	+3.6
1.5	4	6.0	+2	+12.0	+2	+24.0
1.2	1	1.2	+3	+3.6	+3	+10.8
		$25.8 = \Sigma_1$		$+6.6 = \Sigma_2$		$60.6 = \Sigma_3$

$$\text{Area of waterplane} = \frac{1}{3} \times \Sigma_1 \times h \times 2 \qquad h = \frac{18}{6} = 3\,\text{m}$$

$$= \frac{1}{3} \times 25.8 \times 3 \times 2$$

$$= 51.6\,\text{m}^2 \text{ (as before)}$$

$$\Sigma_2 = +6.6$$

The +ve sign shows Centre of Flotation is in aft body

$$\text{Centre of Flotation from amidships} = \frac{\Sigma_2}{\Sigma_1} \times h$$

$$= +\frac{6.6}{25.8} \times 3$$

∴ Centre of Flotation = +0.77 m or 0.77 m aft amidships (as before)

$$I_{)(} = \tfrac{1}{3} \times \Sigma_3 \times h^3 \times 2 = \tfrac{1}{3} \times 60.6 \times 3^3 \times 2$$

$$\therefore\ I_{)(} = 1090.8\,\text{m}^4$$

But

$$I_{LCF} = I_{)(} - A\bar{y} = 1090.8 - (51.6 \times 0.77^2)$$

$$= 1090.8 - 30.6$$

$$= \underline{1060\,\text{m}^4}$$

i.e. very close to previous answer.

With this improved method the levers are much less in value. Consequently the error is decreased when predicting $LCF_{)(}$ and I_{LCF}.

Summary

When using Simpson Rules for second moments of area the procedure should be as follows:

1 Make a sketch from the given information.
2 Use a moment Table and insert values.
3 Using summations obtained in the Table proceed to calculate area, LCF, $I_{)(}$, I_{LCF}, $I_{℄}$, etc.
4 Remember: sketch, table, calculation

Exercise 29

1 A large square has a smaller square cut out of its centre such that the second moment of the smaller square about an axis parallel to one side and passing through the centroid is the same as that of the portion remaining about the same axis. Find what proportion of the area of the original square is cut out.

2 Find the second moment of a square of side 2a about its diagonals.

3 Compare the second moment of a rectangle 40 cm × 30 cm about an axis through the centroid and parallel to the 40 cm side with the second moment about an axis passing through the centroid and parallel to the 30 cm side.

4 An H-girder is built from 5 cm thick steel plate. The central web is 25 cm high and the overall width of each of the horizontal flanges is 25 cm. Find the second moment of the end section about an axis through the centroid and parallel to the horizontal flanges.

5 A ship's waterplane is 36 m long. The half-ordinates, at equidistant intervals, commencing from forward, are as follows:

0, 4, 5, 6, 6, 5 and 4 m respectively.

Calculate the second moment of the waterplane area about the centre line and also about a transverse axis through the centre of flotation.

6 A ship's waterplane is 120 metres long. The half-ordinates at equidistant intervals from forward are as follows:

0, 3.7, 7.6, 7.6, 7.5, 4.6 and 0.1 m respectively.

Calculate the second moment of the waterplane area about the centre line and about a transverse axis through the centre of flotation.

7 A ship of 12 000 tonnes displacement is 150 metres long at the waterline. The half-ordinates of the waterplane at equidistant intervals from forward are as follows:

0, 4, 8.5. 11.6, 12.2, 12.5, 12.5, 11.6, 5.2, 2.4 and 0.3 m respectively.

Calculate the longitudinal and transverse BM's.

8 The half-ordinates of a ship's waterplane at equidistant intervals from forward are as follows:

0, 1.3, 5.2, 8.3, 9.7, 9.8, 8.3, 5.3, and 1.9 m respectively.

If the common interval is 15.9 metres, find the second moment of the waterplane area about the centre line and a transverse axis through the centre of flotation.

9 A ship's waterplane is 120 metres long. The half-ordinates commencing from aft are as follows:

0, 1.3, 3.7, 7.6, 7.6, 7.5, 4.6, 1.8 and 0.1 m respectively.

The spacing between the first three and the last three half-ordinates is half of that between the other half-ordinates. Calculate the second moment of the waterplane area about the centre line and about a transverse axis through the centre of flotation.

10 A ship's waterplane is 90 metres long between perpendiculars. The half-ordinates at this waterplane are as follows:

Station	AP	$\frac{1}{2}$	1	2	3	4	5	$5\frac{1}{2}$	FP
$\frac{1}{2}$ Ords. (m)	0	2	4.88	6.71	7.31	7.01	6.40	2	0.9

Calculate the second moment of the waterplane area about the centre line and also about a transverse axis through the centre of flotation.

Chapter 30
Liquid pressure and thrust. Centres of pressure

Pressure in liquids

When a fluid is in equilibrium the stress across any surface in it is normal to the surface and the pressure intensity at any point of a fluid at rest is the same in all directions. The pressure intensity in a homogeneous liquid at rest under gravity increases uniformly with depth. i.e.

$$P = w \times g \times D$$

where

$$P = \text{Pressure intensity}$$

$$w = \text{Density of the liquid,}$$

$$g = \text{Acceleration due to gravity,}$$

and

$$D = \text{Depth below the surface.}$$

Total Thrust and Resultant Thrust

If the thrust on each element of area of a surface immersed in a fluid is found, the scalar sum of all such thrusts is called the 'Total Thrust' whilst their vector sum is called the 'Resultant Thrust'. When the surface is plane then the Total Thrust is equal to the Resultant Thrust.

To find the Resultant Thrust

In Figure 30.1, G represents the centroid of an area which is immersed, though not necessarily vertical, in a liquid, and Z represents the depth of the centroid of the area below the surface. Let w be the mass density of the liquid. If an element (dA) of the area, whose centroid is Z_1 below the surface, is considered, then:

$$\text{Thrust on the element dA} = \text{Pressure intensity} \times \text{Area}$$

$$= w \times g \times Z_1 \times dA$$

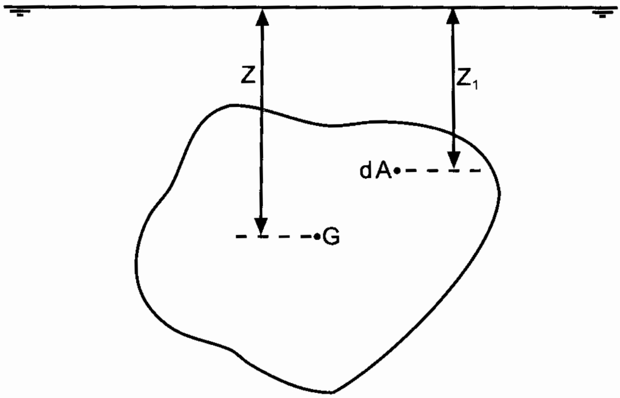

Fig. 30.1

$$\text{Resultant thrust on the whole area are} = \int w \times g \times Z_1\, dA$$

$$= wg \times \int Z_1\, dA$$

but

$$\int Z_1\, dA = Z \cdot A \cdot$$

and

$$w \times g \int Z_1\, dA = w \times g \cdot Z \cdot A \cdot$$

$\therefore$ Resultant thrust = Density × g × Depth of centroid × Area

It should be noted that this formula gives only the magnitude of the resultant thrust. It does not indicate the point at which the resultant thrust may be considered to act.

The centre of pressure

The centre of pressure is the point at which the resultant thrust on an immersed surface may be considered to act. Its position may be found as follows:

(i) For a rectangular lamina immersed with one side in the surface of the liquid. In Figure 30.2, each particle of the strip GH is approximately at the same depth and therefore, the pressure intensity is nearly the same on each particle. Hence the resultant thrust on each strip will act at its mid-point. The resultant of the thrusts on all of the strips in the lamina will act at a point on EF.

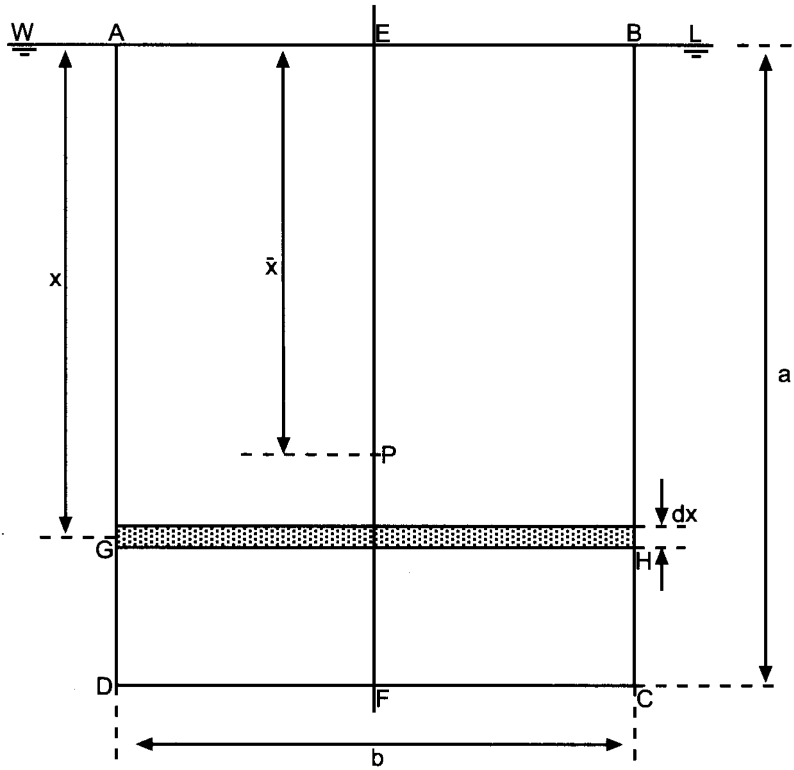

Fig. 30.2

Let w = Mass density of the liquid

The area of the elementary strip = b dx

The depth of the centroid of the strip below the surface is x if the plane of the rectangle is vertical, or x sin θ if the plane is inclined at an angle θ to the horizontal.

$$\text{The thrust on the strip} = w \cdot g \cdot b \cdot x \cdot \sin\theta \cdot dx$$

$$\text{The resultant thrust on the lamina} = \int_O^a w \cdot g \cdot b \cdot x \cdot \sin\theta \cdot dx$$

$$= \frac{w \cdot g \cdot b \cdot a^2}{2} \sin\theta$$

$$= a \cdot b \cdot \times \tfrac{1}{2} \cdot a \cdot \sin\theta \times w \times g$$

= Area × Depth of centroid

× g × Density

$$\text{The moment of the thrust on the strip about AB} = w \cdot g \cdot b \cdot x^2 \cdot \sin\theta \cdot dx$$

$$\text{The moment of the total thrust about AB} = \int_O^a w \cdot g \cdot b \cdot x^2 \cdot \sin\theta$$

$$= w \cdot g \cdot b \cdot \frac{a^3}{3} \cdot \sin\theta$$

Let $\bar{X}$ be the distance of the centre of pressure (P) from AB, then:

$$\bar{X} \times \text{Total thrust} = \text{Total moment about AB}$$

$$\bar{X} \times w \cdot g \cdot b \cdot \frac{a^2}{2} \cdot \sin\theta = w \cdot g \cdot b \frac{a^3}{3} \cdot \sin\theta$$

or

$$\bar{X} = \tfrac{2}{3}\, a \text{ (unless } \sin\theta = 0)$$

(ii) For any Plane Area immersed in a liquid.

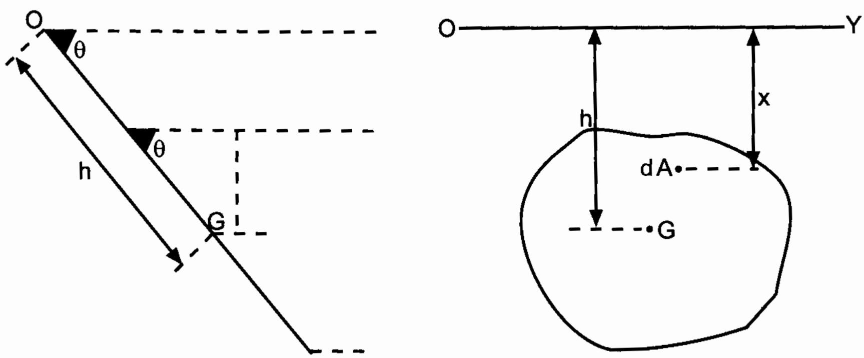

Fig. 30.3

In Figure 30.3, let OY be the line in which the plane cuts the surface of the liquid. Let the plane be inclined at an angle θ to the horizontal.

Let h = the x co-ordinate of the centroid (G), and

let w = the mass density of the liquid.

$$\text{Depth of the element } dA = x \cdot \sin\theta$$

$$\text{Thrust on } dA = w \cdot g \cdot x \cdot \sin\theta \cdot dA$$

$$\text{Moment of thrust about OY} = w \cdot g \cdot x^2 \cdot \sin\theta \cdot dA$$

$$\text{Moment of total thrust about OY} = \int w \cdot g \cdot x^2 \cdot \sin\theta \cdot dA$$

$$= w \cdot g \cdot \sin\theta \cdot \int x^2 \cdot dA$$

$$\text{Total thrust on A} = w \cdot g \cdot A \cdot \text{Depth of centroid}$$

$$= w \cdot g \cdot A \cdot h \cdot \sin\theta$$

$$\text{Moment of total thrust about OY} = w \cdot g \cdot A \cdot h \cdot \sin\theta \cdot \bar{X}$$

$$\therefore\ w \cdot g \cdot A \cdot h \cdot \sin\theta \cdot \bar{X} = w \cdot g \cdot \sin\theta \cdot \int x^2 \cdot dA$$

or

$$\bar{X} = \frac{\int x^2 \cdot dA}{hA} \quad (\text{Unless } \sin\theta = 0)$$

Let I_{OY} be the second moment of the area about OY, then

$$I_{OY} = \int x^2 \cdot dA$$

and

$$\bar{X} = \frac{I_{OY}}{hA}$$

or

$$\bar{X} = \frac{\text{Second moment of area about the waterline}}{\text{First moment of area about the waterline}}$$

Centres of pressure by Simpson's Rules

Using Horizontal Ordinates

Referring to Figure 30.4:

$$\text{Thrust on the element} = w \cdot g \cdot x \cdot y \cdot dx$$

$$\text{Moment of the thrust about OY} = w \cdot g \cdot x^2 \cdot y \cdot dx$$

$$\text{Moment of total thrust about OY} = \int w \cdot g \cdot x^2 \cdot y \cdot dx$$

$$= w \cdot g \int x^2 \cdot y \cdot dx$$

$$\text{Total Thrust} = w \cdot g \cdot A \cdot \text{Depth of centroid}$$

$$= w \cdot g \cdot \int y \cdot dx \cdot \frac{\int x \cdot y \cdot dx}{\int y \cdot dx}$$

$$= w \cdot g \cdot \int x \cdot y \cdot dx$$

$$\text{Moment of total thrust about OY} = \text{Total Thrust} \times \bar{X}$$

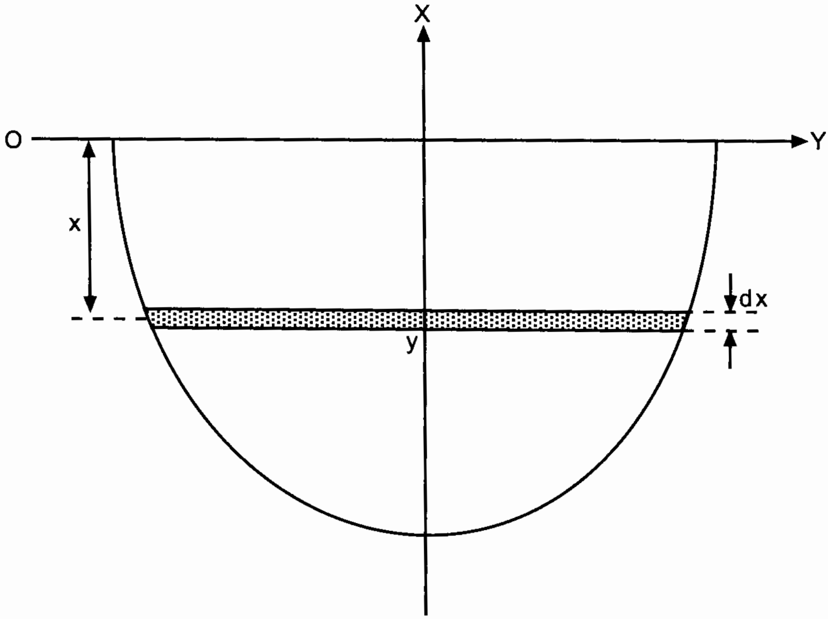

Fig. 30.4

where

$$\bar{X} = \text{Depth of centre of pressure below the surface}$$

$$\therefore \text{ Moment of total thrust about OY} = w \cdot g \cdot \int x \cdot y \cdot dx \times \bar{X}$$

or

$$w \cdot g \cdot \int x \cdot y \cdot dx \times \bar{X} = w \cdot g \cdot \int x^2 \cdot y \cdot dx$$

and

$$\bar{X} = \frac{\int x^2 \cdot y \cdot dx}{\int x \cdot y \cdot dx}$$

The value of the expression $\int x^2 \cdot y \cdot dx$ can be found by Simpson's Rules using values of the product x^2y as ordinates, and the value of the expression $\int x \cdot y \cdot dx$ can be found in a similar manner using values of the product xy as ordinates.

Example 1

A lower hold bulkhead is 12 metres deep. The transverse widths of the bulkhead, commencing at the upper edge and spaced at 3 m intervals, are as follows:

15.4, 15.4, 15.4, 15.3 and 15 m respectively.

Find the depth of the centre of pressure below the waterplane when the hold is flooded to a depth of 2 metres above the top of the bulkhead.

Ord.	*SM*	*Area func.*	*Lever*	*Moment func.*	*Lever*	*Inertia func.*
15.4	1	15.4	0	0	0	0
15.4	4	61.6	1	61.6	1	61.6
15.4	2	30.8	2	61.6	2	123.2
15.3	4	61.2	3	183.6	3	550.8
15.0	1	15.0	4	60.0	4	240.0
		$184.0 = \Sigma_1$		$366.8 = \Sigma_2$		$975.6 = \Sigma_3$

$$\text{Area} = \tfrac{1}{3} \times h \times \Sigma_1 = \tfrac{3}{3} \times 184.0$$
$$= 184 \text{ sq m}$$

Referring to Figure 30.5:

$$CG = \frac{\Sigma_2}{\Sigma_1} \times h$$
$$= \frac{366.8}{184} \times 3$$
$$= 5.98 \text{ m}$$
$$+CD = 2.00 \text{ m}$$
$$\bar{z} = 7.98 \text{ m}$$
$$I_{OZ} = \tfrac{1}{3} \times h^3 \times \Sigma_3 = \tfrac{1}{3} \times 3^3 \times 975.6$$
$$= 8780 \text{ m}^4$$
$$I_{CG} = I_{OZ} - A(CG)^2, \text{ i.e. parallel axis theorem}$$
$$I_{WL} = I_{CG} + A\bar{z}^2$$
$$= I_{OZ} - A(CG^2 - \bar{z}^2)$$
$$I_{WL} = 8780 - 184(5.98^2 - 7.98^2)$$
$$= 13\,928 \text{ m}^4$$
$$\bar{y} = \frac{I_{WL}}{A\bar{z}}$$
$$= \frac{13\,928}{184 \times 7.98}$$
$$\bar{y} = 9.5 \text{ m.}$$

Ans. The Centre of Pressure is 9.5 m below the waterline.

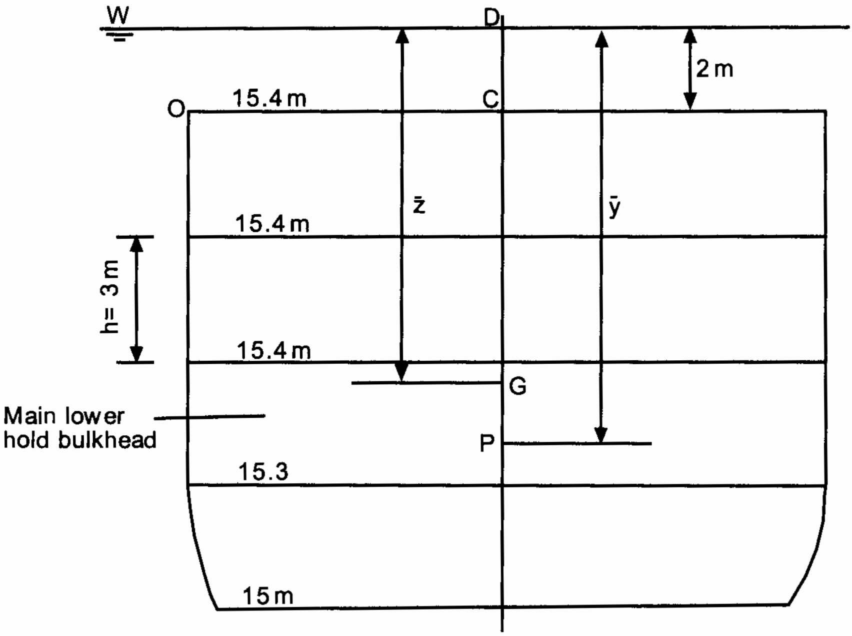

Fig. 30.5

Using vertical ordinates

Referring to Figure 30.6.

$$\text{Thrust on the element} = w \cdot g \cdot \frac{y}{2} \cdot y \cdot dx$$

$$= \frac{w \cdot g \cdot y^2}{2} \cdot dx$$

$$\text{Moment of the thrust about OX} = \frac{w \cdot g \cdot y^2}{2} \cdot dx \cdot \frac{2}{3}\,y$$

$$= \frac{w \cdot g \cdot y^3}{3} \cdot dx$$

$$\text{Moment of total thrust about OX} = \frac{w}{3} \cdot g \cdot \int y^3 \cdot dx$$

$$\text{Total Thrust} = w \cdot g \cdot A \cdot \text{Depth at centre of gravity}$$

$$= w \cdot g \cdot \int y \cdot dx \cdot \frac{\frac{1}{2}\int y^2 \cdot dx}{\int y \cdot dx}$$

$$= \frac{w}{2} \cdot g \cdot \int y^2 \cdot dx$$

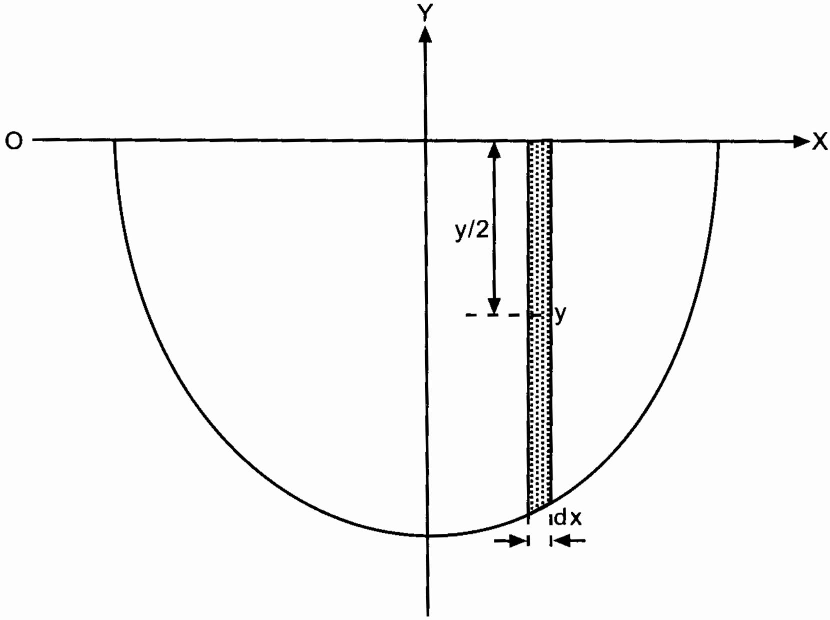

Fig. 30.6

Let $\bar{Y}$ be the depth of the centre of pressure below the surface, then:

$$\text{Moment of total thrust about OX} = \text{Total Thrust} \times \bar{Y}$$

$$\frac{wg}{3}\int y^3 \cdot dx = \frac{wg}{2}\int y^2 \cdot dx \cdot \bar{Y}$$

or

$$\bar{Y} = \frac{\frac{1}{3}\int y^3 \cdot dx}{\frac{1}{2}\int y^2 \cdot dx}$$

The values of the two integrals can again be found using Simpson's Rules.

Example 2

The breadth of the upper edge of a deep tank bulkhead is 12 m. The vertical heights of the bulkhead at equidistant intervals across it are 0, 3, 5, 6, 5, 3 and 0 m respectively. Find the depth of the centre of pressure below the waterline when the tank is filled to a head of 2 m above the top of the tank.

$$\text{Area} = \tfrac{1}{3} \times \text{CI} \times \Sigma_1$$

$$\text{Area} = \tfrac{1}{3} \times 2 \times 68$$

$$= 45\tfrac{1}{3} \text{ sq m}$$

Ord,	*SM*	*Area func.*	*Ord.*	*Moment func.*	*Ord.*	*Inertia func.*
0	1	0	0	0	0	0
3	4	12	3	36	3	108
5	2	10	5	50	5	250
6	4	24	6	144	6	864
5	2	10	5	50	5	250
3	4	12	3	36	3	108
0	1	0	0	0	0	0
		$68 = \Sigma_1$		$316 = \Sigma_2$		$1580 = \Sigma_3$

Referring to Figure 30.7:

$$CG = \frac{\Sigma_2}{\Sigma_1} \times \frac{1}{2}$$

$$= \frac{316}{68} \times \frac{1}{2}$$

$$= 2.324\,\text{m}$$

$$CD = \underline{2.000\,\text{m}}$$

$$DG = \underline{4.324\,\text{m}} = \bar{z}$$

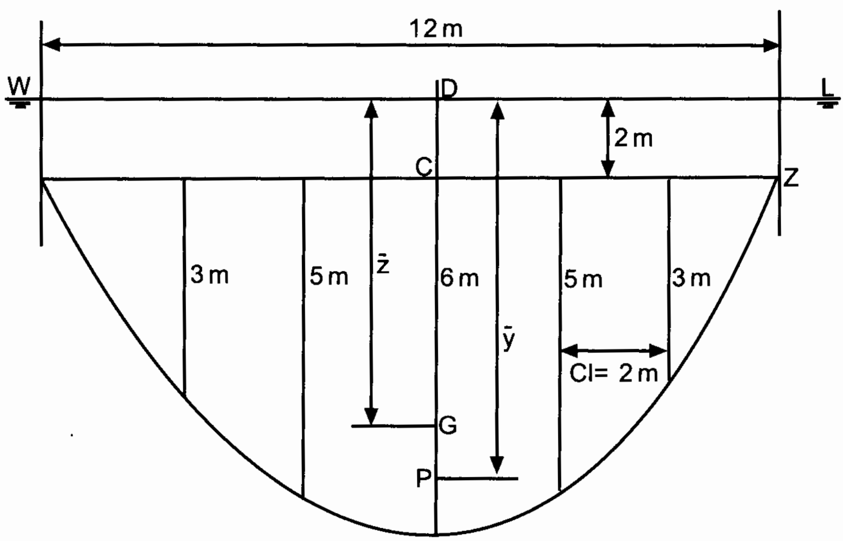

Fig. 30.7

$$I_{OZ} = \tfrac{1}{9} \times CI \times \Sigma_3$$

$$= \tfrac{1}{9} \times 2 \times 1580 = 351\,m^4$$

$$I_{CG} = I_{OZ} - A\,(CG)^2$$

$$I_{WL} = I_{CG} + A\bar{z}^2$$

$$= I_{OZ} - A\,(CG)^2 + A\bar{z}^2$$

$$= I_{OZ} - A\,(CG^2 - \bar{z}^2)$$

$$= 351 - 45.33\,(2.324^2 - 4.324^2)$$

$$I_{WL} = 953.75\,m^4$$

$$\bar{y} = \frac{I_{WL}}{A\bar{z}}$$

$$= \frac{953.75}{45.33 \times 4.324}$$

$$\bar{y} = 4.87\,m$$

Ans. The Centre of Pressure is 4.87 m below the waterline.

Summary

When using Simpson's Rules to estimate the area of a bulkhead under liquid pressure together with the VCG and centre of pressure the procedure should be as follows:

1 Make a sketch from the given information.
2 Make a table and insert the relevant ordinates and multipliers.
3 Calculate the area of bulkhead's plating.
4 Estimate the ship's VCG below the stipulated datum level.
5 Using the parallel axis theorem, calculate the requested centre of pressure.
6 Remember: sketch, table, calculation.

Exercise 30

1 A fore-peak tank bulkhead is 7.8 m deep. The widths at equidistant intervals from its upper edge to the bottom are as follows:

16, 16.6, 17, 17.3, 16.3, 15.3 and 12 m respectively.

Find the load on the bulkhead and the depth of the centre of pressure below the top of the bulkhead when the fore peak is filled with salt water to a head of 1.3 m above the crown of the tank.

2 A deep tank transverse bulkhead is 30 m deep. Its width at equidistant intervals from the top to the bottom is:

20, 20.3, 20.5, 20.7, 18, 14 and 6 m respectively.

Find the depth of the centre of pressure below the top of the bulkhead when the tank is filled to a head of 4 m above the top of the tank.

3 The transverse end bulkhead of a deep tank is 18 m wide at its upper edge. The vertical depths of the bulkhead at equidistant intervals across it are as follows:

0, 3.3, 5, 6, 5, 3.3 and 0 m respectively.

Find the depth of the centre of pressure below the top of the bulkhead when the tank is filled with salt water to a head of 2 m above the top of the bulkhead. Find also the load on the bulkhead.

4 A fore-peak bulkhead is 18 m wide at its upper edge. Its vertical depth at the centre line is 3.8 m. The vertical depths on each side of the centre line at 3 m intervals are 3.5, 2.5 and 0.2 m respectively. Calculate the load on the bulkhead and the depth of the centre of pressure below the top of the bulkhead when the fore-peak tank is filled with salt water to a head of 4.5 m above the top of the bulkhead.

5 The vertical ordinates across the end of a deep tank transverse bulkhead measured downwards from the top at equidistant intervals, are:

4, 6, 8, 9.5, 8, 6 and 4 m respectively.

Find the distance of the centre of pressure below the top of the bulkhead when the tank is filled with salt water.

6 A square plate of side 'a' is vertical and is immersed in water with an edge of its length in the free surface. Prove that the distance between the centres of pressure of the two triangles into which the plate is divided by a diagonal, is $\frac{a\sqrt{13}}{8}$