Chapter 22
Dynamical stability

Dynamical stability is defined as the work done in inclining a ship.

Consider the ship shown in Figure 22.1. When the ship is upright the force 'W' acts upwards through B and downwards through G. These forces act throughout the inclination. b = w.

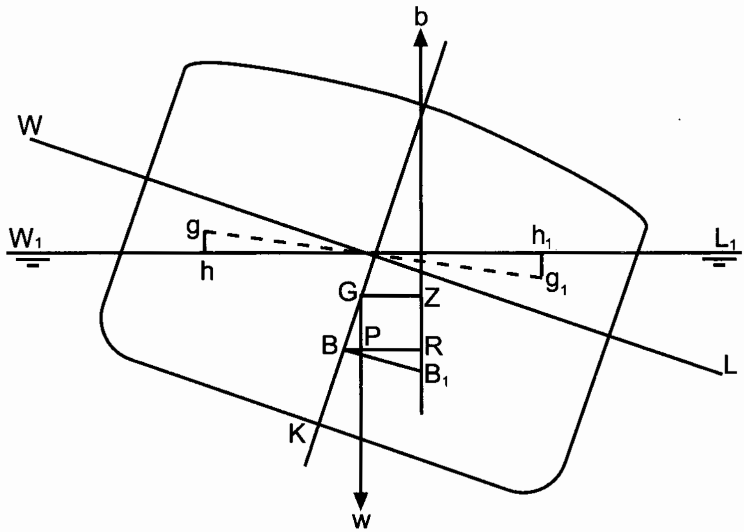

Fig. 22.1

$$\text{Work done} = \text{Weight} \times \text{vertical separation of G and B}$$

or

$$\text{Dynamical stability} = W \times (B_1Z - BG)$$

$$= W \times (B_1R + RZ - BG)$$

$$= W \times \left[\frac{v(gh + g_1h_1)}{V} + PG - BG\right]$$

$$= W \times \left[\frac{v(gh + g_1h_1)}{V} + BG \cos\theta - BG\right]$$

$$\text{Dynamical stability} = W\left[\frac{v(gh + g_1h_1)}{V} - BG(1 - \cos\theta)\right]$$

This is known as *Moseley's formula* for dynamical stability.

If the curve of statical stability for a ship has been constructed the dynamical stability to any angle of heel may be found by multiplying the area under the curve to the angle concerned by the vessel's displacement. i.e.

Dynamical stability = W × Area under the stability curve

The derivation of this formula is as follows:

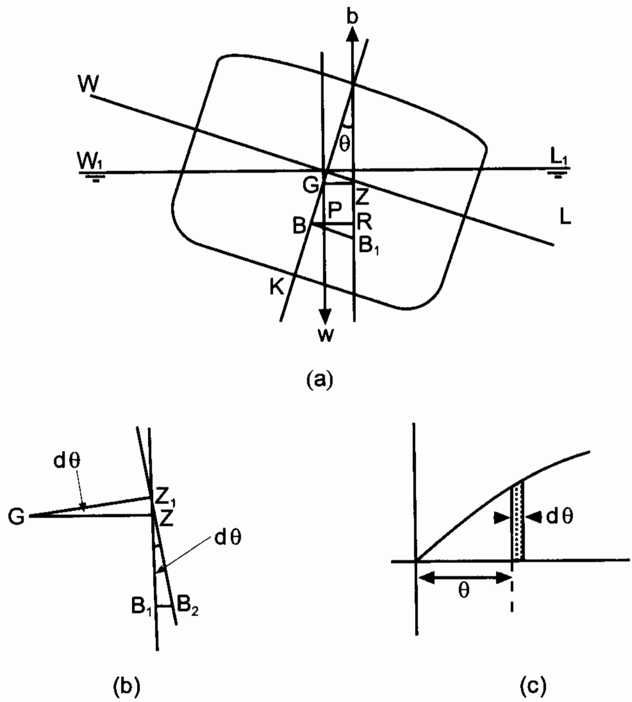

Fig. 22.2

Consider Figure 22.2(a) which shows a ship heeled to an angle θ. Now let the ship be heeled through a further very small angle dθ. The centre of buoyancy B_1 will move parallel to W_1L_1 to the new position B_2 as shown in Figure 22.2(b).

B_2Z_1 is the new vertical through the centre of buoyancy and GZ_1 is the new righting arm. The vertical separation of Z and Z_1 is therefore $GZ \times d\theta$. But this is also the vertical separation of B and G. Therefore the dynamical stability from θ to $(\theta + d\theta)$ is $W \times (GZ \times d\theta)$.

Refer now to Figure 22.2(c) which is the curve of statical stability for the ship. At θ the ordinate is GZ. The area of the strip is $GZ \times d\theta$. But $W \times (GZ \times d\theta)$ gives the dynamical stability from θ to $(\theta + d\theta)$, and this must be true for all small additions of inclination.

$$\therefore \text{ Dynamical stability} = \int_0^\theta W \times GZ \times d\theta$$

$$= W \int_0^\theta GZ\, d\theta$$

Therefore the dynamical stability to any angle of heel is found by multiplying the area under the stability curve to that angle by the displacement.

It should be noted that in finding the area under the stability curve by the use of Simpson's Rules, the common interval must be expressed in *radians*.

$$57.3^\circ = 1 \text{ radian}$$

$$1^\circ = \frac{1}{57.3} \text{ radians}$$

or

$$x^\circ = \frac{x}{57.3} \text{ radians}$$

Therefore to convert degrees to radians simply divide the number of degrees by 57.3.

Example 1

A ship of 5000 tonnes displacement has righting levers as follows:

Angle of heel	10°	20°	30°	40°
GZ (metres)	0.21	0.33	0.40	0.43

Calculate the dynamical stability to 40 degrees heel.

GZ	*SM*	*Functions of area*
0	1	0
0.21	4	0.84
0.33	2	0.66
0.40	4	1.60
0.43	1	0.43
		$3.53 = \Sigma_1$

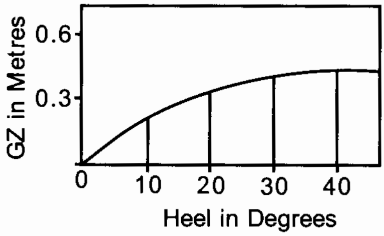

Fig. 22.3

$$h = 10^\circ$$

$$h = \frac{10}{57.3} \text{ radians} = \text{common interval CI}$$

$$\text{The area under the stability curve} = \frac{1}{3} \times \text{CI} \times \Sigma_1$$

$$= \frac{1}{3} \times \frac{10}{57.3} \times 3.53$$

$$= 0.2053 \text{ metre-radians}$$

$$\text{Dynamical stability} = \text{W} \times \text{Area under the stability curve}$$

$$= 5000 \times 0.2053$$

Ans. Dynamical stability = 1026.5 metre tonnes

Example 2

A box-shaped vessel 45 m × 10 m × 6 m is floating in salt water at a draft of 4 m F and A. GM = 0.6 m. Calculate the dynamical stability to 20 degrees heel.

$$\text{BM} = \frac{B^2}{12d} \qquad \text{Displacement} = 45 \times 10 \times 4 \times 1.025 \text{ tonnes}$$

$$= \frac{10 \times 10}{12 \times 4} \qquad \text{Displacement} = 1845 \text{ tonnes}$$

$$\text{BM} = 2.08 \text{ m}$$

Note. When calculating the GZ's 10 degrees may be considered a small angle of heel, but 20 degrees is a large angle of heel, and therefore, the wall-sided formula must be used to find the GZ.

GZ	*SM*	*Products for area*
0	1	0
0.104	4	0.416
0.252	1	0.252
		$0.668 = \Sigma_1$

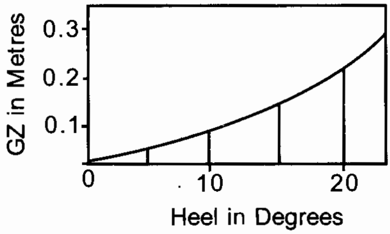

Fig. 22.4

At 10° heel:

$$GZ = GM \times \sin\theta$$
$$= 0.6 \times \sin 10^\circ$$
$$GZ = 0.104\,\text{m}$$

At 20° heel:

$$GZ = (GM + \tfrac{1}{2}\,BM\tan^2\theta)\sin\theta$$
$$= (0.6 + \tfrac{1}{2} \times 2.08 \times \tan^2 20^\circ)\sin 20^\circ$$
$$= (0.6 + 0.138)\sin 20^\circ$$
$$= 0.738 \sin 20^\circ$$
$$GZ = 0.252\,\text{m}$$

$$\text{Area under the curve} = \frac{1}{3} \times CI \times \Sigma_1$$
$$= \frac{1}{3} \times \frac{10}{57.3} \times 0.668$$
$$\text{Area under the curve} = 0.0389 \text{ metre radians}$$
$$\text{Dynamical stability} = W \times \text{Area under the curve}$$
$$= 1845 \times 0.0389$$

Ans. Dynamical stability = 71.77 m tonnes

Exercise 22

1 A ship of 10 000 tonnes displacement has righting levers as follows:

Heel	10°	20°	30°	40°
GZ (m)	0.09	0.21	0.30	0.33

Calculate the dynamical stability to 40 degrees heel.

2 When inclined, a ship of 8000 tonnes displacement has the following righting levers:

Heel	15°	30°	45°	60°
GZ (m)	0.20	0.30	0.32	0.24

Calculate the dynamical stability to 60 degrees heel.

3 A ship of 10 000 tonnes displacement has the following righting levers when inclined:

Heel	0°	10°	20°	30°	40°	50°
GZ (m)	0.0	0.02	0.12	0.21	0.30	0.33

Calculate the dynamical stability to 50 degrees heel.

4 A box-shaped vessel 42 m × 6 m × 5 m, is floating in salt water on an even keel at 3 m draft and has KG = 2 m. Assuming that the KM is constant, calculate the dynamical stability to 15 degrees heel.

5 A box-shaped vessel 65 m × 10 m × 6 m is floating upright on an even keel at 4 m draft in salt water. GM = 0.6 m. Calculate the dynamical stability to 20 degrees heel.

Chapter 23
Effect of beam and freeboard on stability

To investigate the effect of beam and freeboard on stability, it will be necessary to assume the stability curve for a particular vessel in a particular condition of loading. Let Curve A in Figure 23.1 represent the curve of stability for a certain box-shaped vessel whose deck edge becomes immersed at about 17 degrees heel.

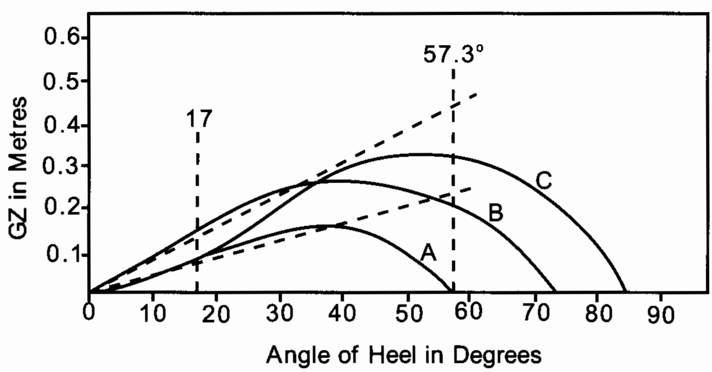

Fig. 23.1

The effect of increasing the beam

Let the draft, freeboard and KG remain unchanged, but increase the beam and consider the effect this will have on the stability curve.

For a ship-shaped vessel BM = I/V, and for a box-shaped vessel $BM = B^2/12d$. Therefore an increase in beam will produce an increase in BM. Hence the GM will also be increased, as will the righting levers at all angles of heel. The range of stability is also increased. The new curve of stability would appear as curve B in Figure 23.1.

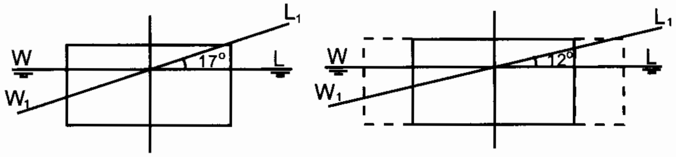

Fig. 23.2

It will be noticed that the curve, at small angles of heel, is much steeper than the original curve, indicating the increase in GM. Also, the maximum GZ and the range of stability have been increased whilst the angle of heel at which the deck edge becomes immersed, has been reduced. The reason for the latter change is shown in Figure 23.2. Angle θ reduces from 17° to 12°.

Figure 23.2(a) represents the vessel in her original condition with the deck edge becoming immersed at about 17 degrees. The increase in the beam, as shown in Figure 23.2(b), will result in the deck edge becoming immersed at a smaller angle of heel. When the deck edge becomes immersed, the breadth of the water-plane will decrease and this will manifest itself in the curve by a reduction in the rate of increase of the GZs with increase in heel.

The effect of increasing the freeboard

Now return to the original vessel. Let the draft, KG, and the beam, remain unchanged, but let the freeboard be increased from f_1 to f_2. The effect of this is shown by Curve C in Figure 23.1.

There will be no effect on the stability curve from the origin up to the angle of heel at which the original deck edge was immersed. When the vessel is now inclined beyond this angle of heel, the increase in the freeboard will cause an increase in the water-plane area and, thus, the righting levers will also be increased. This is shown in Figure 23.2(c), where WL represents the original breadth of the water-plane when heeled x degrees, and WL_1 represents the breadth of the water-plane area for the

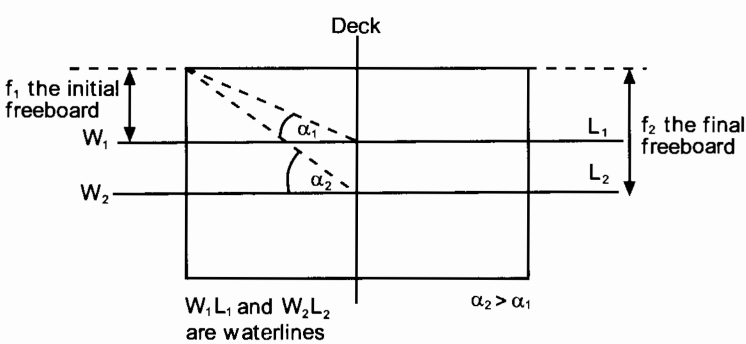

Fig. 23.2(c)

same angle of heel but with the increased freeboard. Thus, the vessel can heel further over before her deck edge is immersed, because $\alpha_2 > \alpha_1$.

From the above it may be concluded that an increase in freeboard has no effect on the stability of the vessel up to the angle of heel at which the original deck edge became immersed, but beyond this angle of heel all of the righting levers will be increased in length. The maximum GZ and the angle at which it occurs will be increased as also will be the range of stability.

Summary

With increased Beam

GM_T and GZ increase.
Range of stability increases.
Deck edge immerses earlier.
KB remains similar.

With increased Freeboard

GM_T and GZ increase.
Range of stability increases.
Deck edge immerses later at greater θ.
KB decreases.

Chapter 24
Angle of loll

When a ship with negative initial metacentric height is inclined to a small angle, the righting lever is negative, resulting in a capsizing moment. This effect is shown in Figure 24.1(a) and it can be seen that the ship will tend to heel still further.

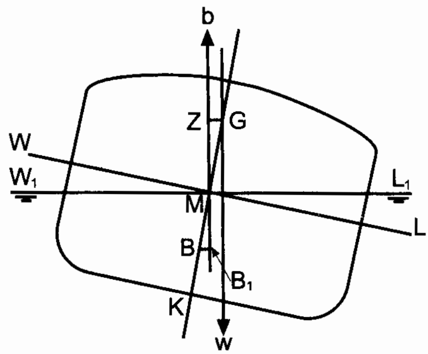

Fig. 24.1 (a)

At a large angle of heel the centre of buoyancy will have moved further out the low side and the force of buoyancy can no longer be considered to act vertically upwards though M, the initial metacentre. If, by heeling still further, the centre of buoyancy can move out far enough to lie vertically under G the centre of gravity, as in Figure 24.1(b), the righting lever and thus the righting moment, will be zero.

The angle of heel at which this occurs is referred to as the *angle of loll* and may be defined as the angle to which a ship with negative initial metacentric height will lie at rest in still water.

If the ship should now be inclined to an angle greater than the angle of loll, as shown in Figure 24.1(c), the righting lever will be positive, giving a moment to return the ship to the angle of loll.

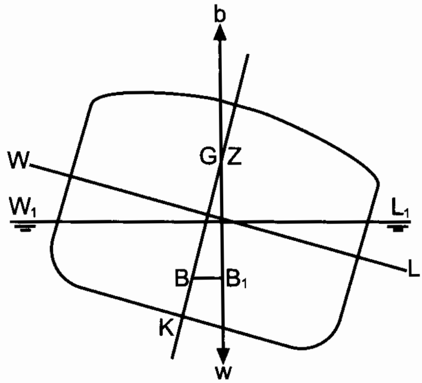

Fig. 24.1 (b)

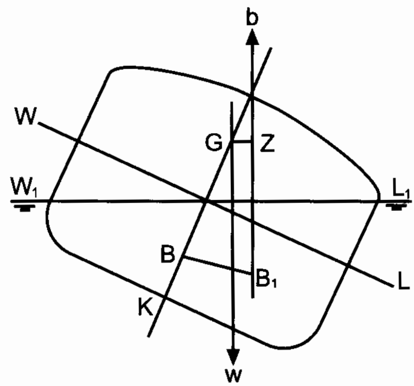

Fig. 24.1 (c)

From this it can be seen that the ship will oscillate about the angle of loll instead of the upright.

The curve of statical stability for a ship in this condition of loading is illustrated in Figure 24.2. Note from the figure that the GZ at the angle of loll is zero. At angles of heel less than the angle of loll the righting levers are negative, whilst beyond the angle of loll the righting levers are positive up to the angle of vanishing stability.

Note how the range of stability in this case is measured from the angle of loll and not from the 'o–o' axis.

To calculate the angle of loll

When the vessel is 'wall-sided' between the upright and inclined waterlines, the GZ may be found using the formula:

$$GZ = \sin\theta(GM + \tfrac{1}{2}\,BM\tan^2\theta)$$

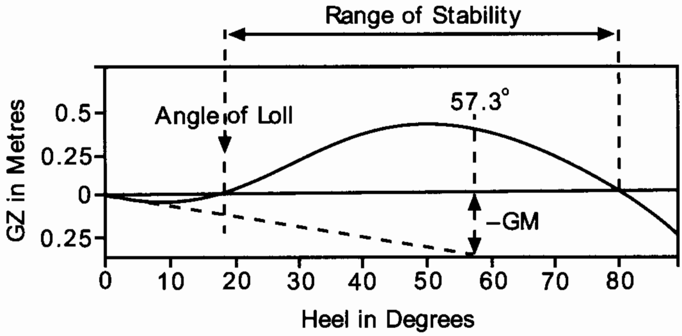

Fig. 24.2

At the angle of loll:

$$GZ = 0$$

$$\therefore \text{ either } \sin\theta = 0$$

or

$$(GM + \tfrac{1}{2}\,BM\tan^2\theta) = 0$$

If

$$\sin\theta = 0$$

then

$$\theta = 0$$

But then angle of loll cannot be zero, therefore:

$$(GM + \tfrac{1}{2}\,BM\tan^2\theta) = 0$$

$$\tfrac{1}{2}\,BM\tan^2\theta = -GM$$

$$BM\tan^2\theta = -2\,GM$$

$$\tan^2\theta = \frac{-2\,GM}{BM}$$

$$\tan\theta = \sqrt{\frac{-2\,GM}{BM}}$$

The angle of loll is caused by a negative GM, therefore:

$$\tan\theta = \sqrt{\frac{-2(-GM)}{BM}}$$

or

$$\tan\theta = \sqrt{\frac{2\,GM}{BM}}$$

where

$$\theta = \text{the angle of loll,}$$
$$GM = \text{a negative initial metacentric height, and}$$
$$BM = \text{the BM when upright.}$$

Example

Will a homogeneous log 6 m × 3 m × 3 m and relative density 0.4 float in fresh water with a side perpendicular to the waterline? If not, what will be the angle of loll?

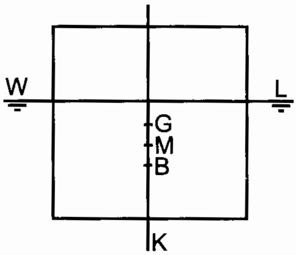

Fig. 24.3

Since the log is homogeneous the centre of gravity must be at half-depth, i.e. KG = 1.5 m.

$$\frac{\text{Draft of log}}{\text{Depth of log}} = \frac{\text{SG of log}}{\text{SG of water}}$$

$$\text{Draft of log} = \frac{3 \times 0.4}{1}$$

$$d = 1.2\text{ m}$$

$$KB = \frac{1}{2}\text{ draft}$$

$$KB = 0.6\text{ m}$$

$$BM = B^2/12d$$

$$= \frac{3 \times 3}{12 \times 1.2}$$

$$BM = 0.625\text{ m}$$
$$+$$
$$KB = 0.600\text{ m}$$
$$KM = 1.225\text{ m}$$
$$KG = 1.500\text{ m}$$
$$GM = -0.275\text{ m}$$

Therefore the log is unstable and will take up an angle of loll.

$$\tan\theta = \sqrt{\frac{2\,GM}{BM}}$$

$$= \sqrt{\frac{0.55}{0.625}} = 0.9381$$

$$\theta = 43^\circ\ 10'$$

Ans. The angle of loll = 43° 10′

Question: What exactly is angle of list and angle of loll? List the differences/characteristics.

Angle of list

'G', the centroid of the loaded weight, has *moved off the centre line* due to a shift of cargo or bilging effects, say to the port side.

GM is positive, i.e. 'G' is below 'M'. In fact GM will *increase* at the angle of list compared to GM when the ship is upright. The ship is in *stable equilibrium*.

In still water conditions the ship will remain at this *fixed* angle of heel. She will list to one side only, that is the same side as movement of weight.

In heavy weather conditions the ship will roll about this angle of list, say 3° P, but will not stop at 3° S. See comment below.

To bring the ship back to upright, load weight on the other side of the ship, for example if she lists 3° P add weight onto starboard side of ship.

Angle of loll

KG = KM so *GM is zero*. 'G' remains *on the centre line* of the ship.

The ship is in *neutral equilibrium*. She is in a *more dangerous situation* than a ship with an angle of list, because once 'G' goes above 'M' she will capsize.

Angle of loll may be *3° P or 3° S* depending upon external forces such as wind and waves acting on her structure. She may suddenly flop over from 3° P to 3° S and then back again to 3° P.

To improve this condition 'G' must be brought below 'M'. This can be done by moving weight downwards towards the keel, adding water ballast in double-bottom tanks or removing weight above the ship's 'G'. Beware of free surface effects when moving, loading, and discharging liquids.

With an angle of list or an angle of loll the calculations must be carefully made *prior* to any changes in loading being made.

Exercise 24

1 Will a homogeneous log of square cross-section and relative density 0.7 be stable when floating in fresh water with two opposite sides parallel to the waterline? If not, what will be the angle of loll?

2 A box-shaped vessel 30 m × 6 m × 4 m floats in salt water on an even keel at 2 m, draft F and A. KG = 3 m. Calculate the angle of loll.

3 A ship is upright and is loaded with a full cargo of timber with timber on deck. During the voyage the ship develops a list, even though stores, fresh water and bunkers have been consumed evenly from each side of the centre line. Discuss the probable cause of the list and the method which should be used to bring the ship to the upright.

4 A ship loaded with a full cargo of timber and timber on deck is alongside a quay and has taken up an angle of loll away from the quay. Describe the correct method of discharging the deck cargo and the precautions which must be taken during this process.