Chapter 19
Combined list and trim

When a problem involves a change of both list and trim, the two parts must be treated quite separately. It is usually more convenient to tackle the trim part of the problem first and then the list, but no hard and fast rule can be made on this point.

Example 1

A ship of 6000 tonnes displacement has KM = 7 m, KG = 6.4 m, and MCT 1 cm = 120 tonnes m. The ship is listed 5 degrees to starboard and trimmed 0.15 m by the head. The ship is to be brought upright and trimmed 0.3 m by the stern by transferring oil from No. 2 double bottom tank to No. 5 double bottom tank. Both tanks are divided at the centre line and their centres of gravity are 6 m out from the centre line. No. 2 holds 200 tonnes of oil on each side and is full. No. 5 holds 120 tonnes on each side and is empty. The centre of gravity of No. 2 is 23.5 m forward of amidships and No. 5 is 21.5 m aft of amidships. Find what transfer of oil must take place and give the final distribution of the oil. (Neglect the effect of free surface on the GM.) Assume that LCF is at amidships.

(a) To bring the ship to the required trim

$$\text{Present trim} = 0.15\text{ m by the head} \circlearrowleft$$

$$\text{Required trim} = \underline{0.30}\text{ m by the stern} \circlearrowright$$

$$\text{Change of trim} = 0.45\text{ m by the stern} \circlearrowright$$

$$= 45\text{ cm by the stern} \circlearrowright$$

$$\text{Trim moment} = \text{Change of trim} \times \text{MCT 1 cm}$$

$$= 45 \times 120$$

$$\text{Trim moment} = 5400\text{ tonnes m by the stern} \circlearrowright$$

Let 'w' tonnes of oil be transferred aft to produce the required trim.

$$\therefore \text{ Trim moment} = w \times d$$
$$= 45w \text{ tonnes m}$$
$$\therefore 45w = 5400$$
$$w = 120 \text{ tonnes}$$

From this it will be seen that, if 120 tonnes of oil is transferred aft, the ship will then be trimmed 0.30 m by the stern.

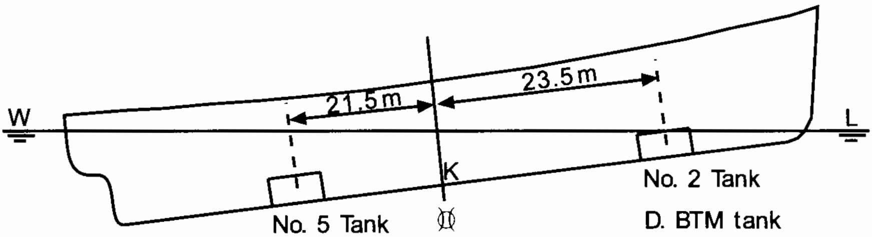


Fig. 19.1 (a)

(b) To bring the ship upright

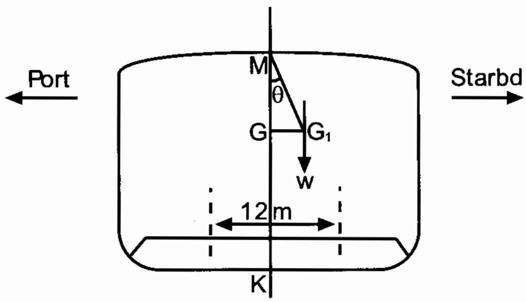


Fig. 19.1 (b). Looking forward

$$KM = 7.0 \text{ m}$$
$$KG = -6.4 \text{ m}$$
$$GM = 0.6 \text{ m}$$

In triangle GG_1M,

$$GG_1 = GM \times \tan\theta$$
$$= 0.6 \times \tan 5^\circ$$
$$GG_1 = 0.0525 \text{ m}$$

Let 'x' tonnes of oil be transferred from starboard to port.

$$\text{Moment to port} = x \times d$$
$$= 12x \text{ tonnes m}$$

$$\text{Initial moment to starboard} = W \times GG_1$$
$$= 6000 \times 0.0525$$
$$= 315 \text{ tonnes m}$$

But if the ship is to complete the operation upright:

$$\text{Moment to starboard} = \text{Moment to port}$$

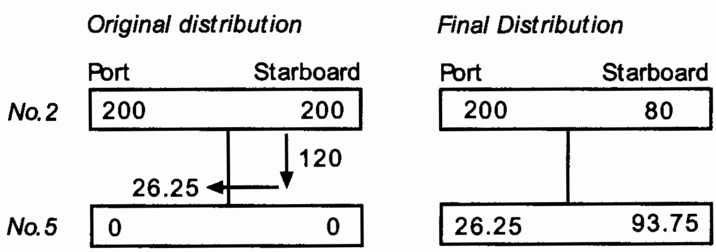
or

$$315 = 12x$$
$$x = 26.25 \text{ tonnes}$$

The ship will therefore be brought upright by transferring 26.25 tonnes from starboard to port.

From this it can be seen that, to bring the ship to the required trim and upright, 120 tonnes of oil must be transferred from forward to aft and 26.25 tonnes from starboard to port. This result can be obtained by taking 120 tonnes from No. 2 starboard and by putting 93.75 tonnes of this oil in No. 5 starboard and the remaining 26.25 tonnes in No. 5 Port tank.

The distributions would then be as follows:



Note. There are, of course, alternative methods by which this result could have been obtained, but in each case a total of *120 tonnes* of oil must be transferred aft and *26.25 tonnes* must be transferred from starboard to port.

Exercise 19

1 A tanker has displacement of 10 000 tonnes, KM = 7 m, KG = 6.4 m and MCT 1 cm = 150 tonnes m. There is a centre line bulkhead in both No. 3 and No. 8 tanks. Centre of gravity of No. 3 tank is 20 m forward of the centre of flotation and the centre of gravity of No. 8 tank is 30 m aft of the centre of flotation. The centre of gravity of all tanks is 5 m out from the centre line. At present the ship is listed 4 degrees to starboard and trimmed 0.15 m by the head. Find what transfer of oil must take place if the ship is to complete upright and trimmed 0.3 m by the stern.

2 A ship of 10 000 tonnes displacement is listed 5 degrees to port and trimmed 0.2 m by the head. KM = 7.5 m, KG 6.8 m, and MCT 1 cm = 150 tonnes m. Centre of flotation is amidships. No.1 double bottom tank is divided at the centre line, each side holds 200 tonnes of oil and the tank is full. No. 4 double bottom tank is similarly divided each side having a capacity of 150 tonnes, but the tank is empty. The centre of gravity of No. 1 tank is 45 m forward of amidships and the centre of gravity of No. 4 tank is 15 m aft of amidships. The centre of gravity of all tanks is 5 m out from the centre line. It is desired to bring the ship upright and trimmed 0.3 m by the stern by transferring oil. If the free surface effect on GM be neglected, find what transfer of oil must take place and also the final distribution of the oil.

3 A ship of 6000 tonnes displacement, KG = 6.8 m., is floating upright in salt water, and the draft is 4 m F and 4.3 m A. KM = 7.7 m TPC = 10 tonnes. MCT 1 cm = 150 tonnes m. There is a locomotive to discharge from No. 2 lower hold (KG = 3 m, and centre of gravity 30 m forward of the centre of flotation which is amidships). If the weight of the locomotive is 60 tonnes and the height of the derrick head is 18 m above the keel and 20 m out from the centre line when plumbing overside, find the maximum list during the operation and the drafts after the locomotive has been discharged. Assume KM is constant.

4 A ship displaces 12 500 tonnes, is trimmed 0.6 m by the stern and listed 6 degrees to starboard. MCT 1 cm 120 tonnes m, KG 7.2 m, KM 7.3 m No. 2 and No. 5 double bottom tanks are divided at the centre line. Centre of gravity of No. 2 is 15 m forward of the centre of flotation and centre of gravity of No. 5 is 12 m abaft centre of flotation. Centre of gravity of all tanks is 4 m out from the centre line. The ship is to be brought upright and on to an even keel by transferring oil from aft to forward, taking equal quantities from each side of No. 5. Find the amounts of oil to transfer.

Chapter 20
Calculating the effect of free surface of liquids (FSE)

The effect of free surface of liquids on stability was discussed in general terms in Chapter 7, but the problem will now be studied more closely and the calculations involved will be explained.

When a tank is partially filled with a liquid, the ship suffers a virtual loss in metacentric height which can be calculated by using the formula:

$$\text{FSE} = \text{Virtual loss of GM} = \frac{i}{W} \times \rho \times \frac{1}{n^2} \text{ metres} \qquad \text{(I)}$$

where

i = the second moment of the free surface about the centre line, in m^4
w = the ship's displacement, in tonnes
ρ = the density of the liquid in the tank, in tonnes/cu. m
n = the number of the longitudinal compartments into which the tank is subdivided
$i \times p$ = free surface moment, in tonnes m.

The derivation of this formula is as follows:

The ship shown in Figure 20.1(a) has an undivided tank which is partially filled with a liquid.

When the ship is inclined, a wedge of the liquid in the tank will shift from the high side to the low side such that its centre of gravity shifts from g to g_1. This will cause the centre of gravity of the ship to shift from G to G_1, where

$$GG_1 = \frac{w \times gg_1}{W}$$

Let

$$\rho_1 = \text{density of the liquid in the tank}$$

and

$$\rho_2 = \text{density of the water in which the ship floats}$$

then

$$GG_1 = \frac{v \times \rho_1 \times gg_1}{V \times \rho_2}$$

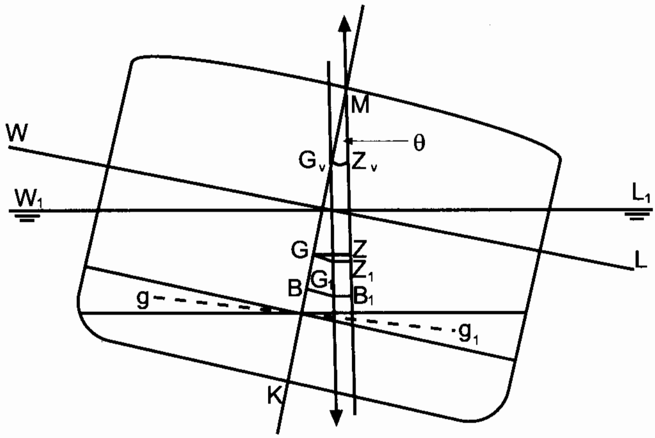


Fig. 20.1 (a)

Had there been no free surface when the ship inclined, the righting lever would have been GZ. But, due to the liquid shifting, the righting lever is reduced to G_1Z_1 or G_vZ_v. The virtual reduction of GM is therefore GG_v.

For a small angle of heel:

$$GG_1 = GG_v \times \theta$$

$$\therefore\ GG_v \times \theta = \frac{v \times gg_1 \times \rho_1}{V \times \rho_2}$$

or

$$GG_v = \frac{v \times gg_1 \times \rho_1}{V \times \theta \times \rho_2}$$

From the proof of BM = I/V, $I \times \theta = v \times gg_1$

Let i = the second moment of the free surface about the centre line.

Then

$$GG_v = \frac{i \times \rho_1}{V \times \rho_2}$$

This is the formula to find the virtual loss of GM due to the free surface effect in an undivided tank.

Now assume that the tank is subdivided longitudinally into 'n' compartments of equal width as shown in Figure 20.1(b).

Let

l = length of the tank, and

b = breadth of the tank.

The breadth of the free surface in each compartment is thus b/n, and the second moment of each free surface is given by $\frac{l \times (b/n)^3}{12}$

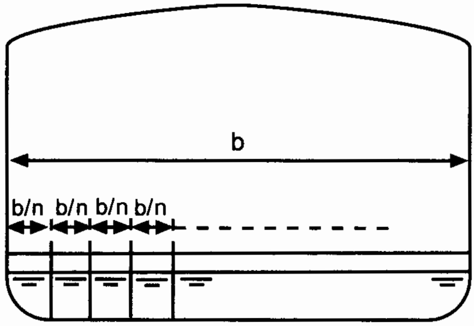


Fig. 20.1(b)

GG_v = virtual loss of GM for one compartment multiplied by the number of compartments.

$$= \frac{i \times \rho_1}{V \times \rho_2} \times n$$

where

i = the second moment of the water-plane area in one compartment about the centre line.

$$= \frac{l \times (b/n)^3 \times \rho_1}{12 \times V \times \rho_2} \times n$$

$$= \frac{l \times b^3 \times \rho_1}{12 \times V \times n^3 \times \rho_2} \times n$$

or

$$GG_v = \frac{i}{V} \times \frac{\rho_1}{\rho_2} \times \frac{1}{n^2}$$

where

i = the second moment of the water-plane area of the whole tank about the centre line.

But

$$W = V \times \rho_2$$

so

$$GG_v = \frac{i}{W} \times \rho_1 \times \frac{1}{n^2}$$

as shown in equation (I).

This is the formula to find the virtual loss of GM due to the free surface effect in a tank which is subdivided longitudinally.

From this formula it can be seen that, when a tank is subdivided longitudinally, the virtual loss of GM for the undivided tank is divided by the square of the number of compartments into which the tank is divided. Also note that the actual weight of the liquid in the tank will have no effect whatsoever on the virtual loss of GM due to the free surface.

For a rectangular area of free surface, the second moment to be used in the above formula can be found as follows:

$$i = \frac{LB^3}{12}$$

where

L = the length of the free surface, and

B = the total breadth of the free surface, ignoring divisions.

Note. Transverse subdivisions in partially filled tanks (slack tanks) *do not* have any influence on reducing free surface effects.

However, fitting longitudinal bulkheads *do* have a very effective influence in reducing this virtual loss in GM.

Example 1

A ship of 8153.75 tonnes displacement has KM = 8 m, KG = 7.5 m, and has a double bottom tank 15 m × 10 m × 2 m which is full of salt water ballast. Find the new GM if this tank is now pumped out till half empty.

Note. The mass of the water pumped out will cause an actual rise in the position of the ship's centre of gravity and the free surface created will cause a virtual loss in GM. There are therefore two shifts in the position of the centre of gravity to consider.

In Figure 20.2 the shaded portion represents the water to be pumped out with its centre of gravity at position g. The original position of the ship's centre of gravity is at G.

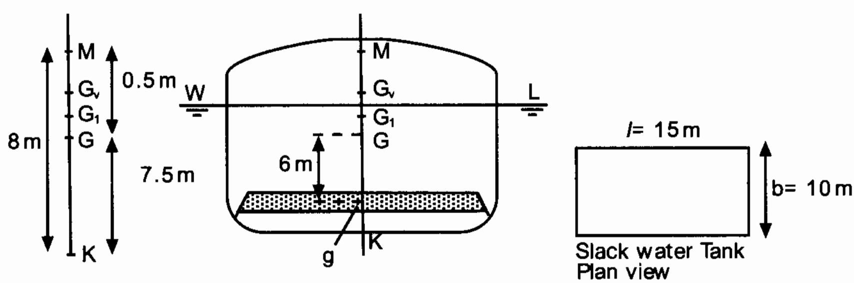


Fig. 20.2

Let GG_1 represent the actual rise of G due to the mass discharged.

$$\text{The mass of water discharged (w)} = 15 \times 10 \times 1 \times 1.025 \text{ tonnes}$$

$$w = 153.75 \text{ tonnes}$$

$$W_2 = W_1 - w = 8153.75 - 153.75$$

$$= 8000 \text{ tonnes}$$

$$GG_1 = \frac{w \times d}{W_2}$$

$$= \frac{153.75 \times 6}{8000}$$

$$\underline{GG_1 = 0.115\text{ m}}$$

Let G_1G_v represent the virtual loss of GM due to free surface or rise in G_1. Then:

$$G_1G_v = \frac{i}{W} \times \rho_1 \times \frac{1}{n^2}$$

as per equation (I)

$$n = 1$$

$$\therefore\ G_1G_v = \frac{i}{W_2} \times \rho_{SW}$$

or

$$G_1G_v = \frac{lb^3}{12} \times \frac{\rho_{SW}}{W_2} \times \frac{1}{n^2}$$

$$\text{Loss in GM} = \text{FSE}$$

or

$$G_1G_v = \frac{15 \times 10^3 \times 1.025}{12 \times 8000}$$

$$\underline{G_1G_v = 0.160\text{ m}} \uparrow$$

$$\text{Old KM} = 8.000\text{ m}$$

$$\text{Old KG} = 7.500\text{ m}$$

$$\text{Old GM} = 0.500\text{ m}$$

$$\text{Actual rise of G} = 0.115\text{ m}$$

$$0.385\text{ m} = GM_{solid}$$

$$G_1G_v = \text{Virtual rise of G} = 0.160\text{ m} \uparrow$$

$$= 0.225\text{ m}$$

Ans. New GM = 0.225 m = GM_{fluid}

Hence G_1 has risen due to the discharge of the ballast water (loading change) and has also risen due to Free Surface Effects.

Be aware that in some cases these two rises of G do not take G above M thereby making the ship unstable.

Example 2

A ship of 6000 tonnes displacement, floating in salt water, has a double bottom tank 20 m × 12 m × 2 m which is divided at the centre line and is partially filled with oil of relative density 0.82. Find the virtual loss of GM due to the free surface of the oil.

$$\text{Virtual loss of GM} = \frac{i}{W} \times \rho_{oil} \times \frac{1}{n^2}$$

$$= \frac{lb^3}{12} \times \rho_{oil} \times \frac{1}{W} \times \frac{1}{n^2}$$

$$= \frac{20 \times 12^3}{12 \times 6000} \times 0.820 \times \frac{1}{2^2}$$

Ans. Virtual loss of GM = 0.098 metres

Example 3

A ship of 8000 tonnes displacement has KM 7.5 m, and KG 7.0 m. A double bottom tank is 12 m long, 15 m wide and 1 m deep. The tank is divided longitudinally at the centre line and both sides are full of salt water. Calculate the list if one side is pumped out until it is half empty. (See Fig. 20.3.)

$$\text{Mass of water discharge} = l_{DB} \times b_{DB} \times \frac{d_{DB}}{2} \times \rho_{SW}$$

$$\text{Mass of water discharged} = 12 \times 7.5 \times 0.5 \times 1.025$$

$$w = 46.125 \text{ tonnes}$$

$$\text{Vertical shift of G(GG}_1) = \frac{w \times d}{W_2}$$

$$= \frac{46.125 \times 6.25}{8000 - 46.125} = \frac{46.125 \times 6.25}{7953.875}$$

$$= 0.036 \text{ metres}$$

$$\text{Horizontal shift of G(G}_V\text{G}_2) = \frac{w \times d}{W - w}$$

$$= \frac{46.125 \times 3.75}{7953.875}$$

$$= \underline{0.022 \text{ metres}}$$

$$\text{Virtual loss of GM(G}_1\text{G}_V) = \frac{l \times b_2^3}{12} \times \frac{\rho_{SW}}{W_2} \times \frac{1}{n^2}$$

$$= \frac{12 \times 7.5^3}{12} \times \frac{1.025}{7953.875} \times \frac{1}{1^2}$$

$$= 0.054 \text{ metres}$$

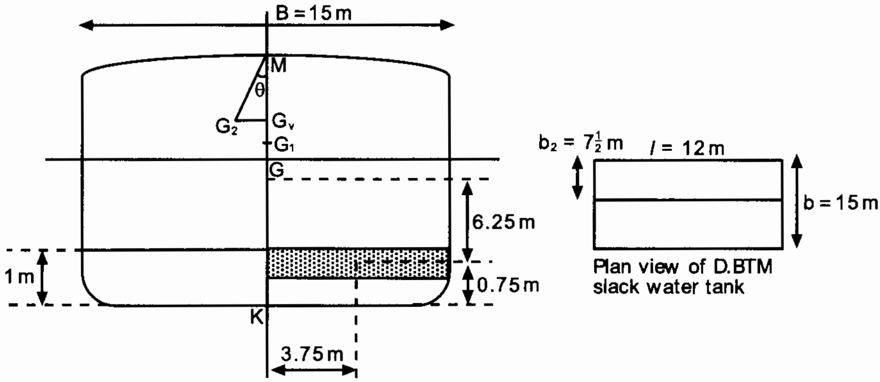


Fig. 20.3

$$\text{KM} = 7.500 \text{ metres}$$

$$\text{Original KG} = \underline{7.000 \text{ metres}}$$

$$\text{Original GM} = 0.500 \text{ metres} = \text{GM}_{\text{solid}}$$

$$\text{Vertical rise of G(GG}_1) = \underline{0.036 \text{ metres}} \uparrow$$

$$\text{G}_1\text{M} = 0.464 \text{ metres} = \text{G}_1\text{M}_{\text{solid}}$$

$$\text{Virtual loss of GM(G}_1\text{G}_v) = 0.054 \text{ metres}$$

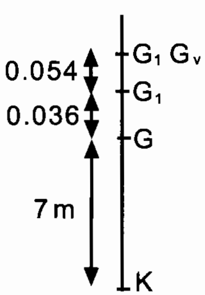
$$\underline{\text{New GM(G}_v\text{M}) = 0.410 \text{ metres} = \text{GM}_{\text{fluid}}}$$

In triangle G_vG_2M:

$$\tan\theta = \frac{G_2G_v}{G_vM}$$

$$= \frac{0.022}{0.410} = 0.0536$$

Ans. List = 3° 04′



Stability factors
Not to scale

So again G has risen due to discharging water ballast. It has also risen due to free-surface effects. A further movement has caused G to move off the centreline and has produced this angle of list of 3° 04′.

The following worked example shows the effect of subdivisions in slack tanks in relation to free surface effects (FSE):

Question: A ship has a displacement of 3000 tonnes. On the vessel is a rectangular double-bottom tank 15 m long and 8 m wide. This tank is partially filled with ballast water having a density of 1.025 t/m^3.

If the GM_T without free surface effects is 0.18 m calculate the virtual loss in GM_T and the final GM_T when the double bottom tank has:

(a) no divisional bulkheads fitted;
(b) one transverse bulkhead fitted at mid-length;
(c) one longitudinal bulkhead fitted on ℄ of the tank;
(d) two longitudinal bulkheads fitted giving three equal divisions.

Answer

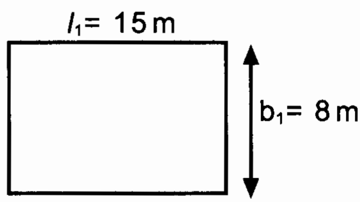


Fig. 20.4(a)

$$\text{FSE} = \text{virtual loss in } GM_T \text{ or rise in } G = \frac{I \times \rho_{SW}}{W}$$

$$= \frac{l \times b_1^3 \times \rho_{SW}}{12 \times W} \quad \text{(see Fig. 20.4(a))}$$

$$\therefore \underline{\text{virtual loss in } GM_T} = \frac{15 \times 8^3 \times 1.025}{3000 \times 12}$$

$$= \underline{0.02187\,\text{m}} \uparrow$$

$$\therefore GM_T \text{ finally} = 0.1800 - 0.2187$$

$$= -\underline{0.0387\,\text{m}} \uparrow$$

i.e. unstable ship!!

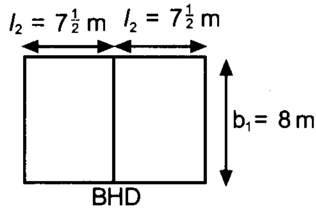


Fig. 20.4(b)

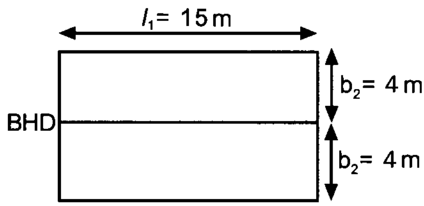


Fig. 20.4(c)

$$\text{FSE} = \text{virtual loss in GM}_T \text{ or rise in G} = \frac{2 @ l_2 \times b_1^3}{12 \times W} \times \rho_{SW} \quad \text{(see Fig. 20.4(b))}$$

$$\therefore \text{ virtual loss} = \frac{2 \times 7.5 \times 8^3 \times 1.025}{12 \times 3000}$$

$$= 0.2187\,\text{m} \uparrow$$

This is same answer as for part (a). Consequently it can be concluded that fitting transverse divisional bulkheads in tanks does not reduce the free surface effects. Ship is still unstable!!

$$\text{FSE} = \text{vertical loss in GM}_T \text{ or rise in G} = \frac{2 @ l_1 b_2^3}{12 \times W} \rho_{SW}$$

$$\therefore \text{ virtual loss in GM}_T = \frac{2 \times 15 \times 4^3 \times 1.025}{12 \times 3000} \quad \text{(see Fig. 20.4(c))}$$

$$= 0.0547\,\text{m} \uparrow \text{ i.e. } \tfrac{1}{4} \text{ of answer to part (a)}$$

Hence

$$\text{final GM}_T = 0.1800 - 0.0547\,\text{m} = +0.1253\,\text{m Ship is stable.}$$

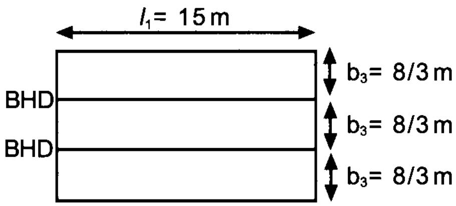


Fig. 20.4(d)

GM_T is now +ve, but below the minimum GM_T of 0.15 m that is allowable by D.Tp. regulations.

$$FSE = \text{virtual loss in } GM_T \text{ or rise in } G = \frac{3 @ l_1 \times b_3^3}{12 \times W} \times \rho_{SW} \quad \text{(see Fig. 20.4(d))}$$

$$\therefore \text{ Virtual loss in } GM_T = \frac{3 \times 15 \times \left(\frac{8}{3}\right)^3 \times 1.025}{12 \times W}$$

$$= 0.0243 \text{ m} \uparrow \quad \text{i.e. } \tfrac{1}{9} \text{ of answer to part (a)}$$

Hence

final $GM_T = 0.1800 - 0.0243 = +0.1557$ m ship is stable.

Ship is stable and above D.Tp. minimum GM_T value of 0.15 m.

So longitudinal divisional bulkheads (watertight or wash-bulkheads) are effective. They cut down rapidly the loss in GM_T. Note the $1/n^2$ law where n is the number of equal divisions made by the longitudinal bulkheads.

Free surface effects therefore depend on:

(I) density of slack liquid in the tank;
(II) ship's displacement in tonnes;
(III) dimensions and shape of the slack tanks;
(IV) bulkhead subdivision within the slack tanks.

The longitudinal divisional bulkheads referred to in examples in this chapter need not be absolutely watertight; they could have openings in them. Examples on board ship are the centreline wash bulkhead in the Fore Peak tank and in Aft Peak tank.

Exercise 20

1 A ship of 10 000 tonnes displacement is floating in dock water of density 1024 kg per cu. m, and is carrying oil of relative density 0.84 in a double-bottom tank. The tank is 25 m long, 15 m wide, and is divided at the centre line. Find the virtual loss of GM due to this tank being slack.

2 A ship of 6000 tonnes displacement is floating in fresh water and has a deep tank (10 m × 15 m × 6 m) which is undivided and is partly filled with nut oil of relative density 0.92. Find the virtual loss of GM due to the free surface.

3 A ship of 8000 tonnes displacement has KG = 3.75 m, and KM = 5.5 m. A double-bottom tank 16 m × 16 m × 1 m is subdivided at the centre line and is full of salt water ballast. Find the new GM if this tank is pumped out until it is half empty.

4 A ship of 10 000 tonnes displacement, KM 6 m, KG 5.5 m, is floating upright in dock water of density 1024 kg per cu. m. She has a double bottom tank 20 m × 15 m which is subdivided at the centre line and is partially filled with oil of relative density 0.96. Find the list if a mass of 30 tonnes is now shifted 15 m transversely across the deck.

5 A ship is at the light displacement of 3000 tonnes and has KG 5.5 m, and KM 7.0 m. The following weights are then loaded:

5000 tonnes of cargo KG 5 m
2000 tonnes of cargo KG 10 m
700 tonnes of fuel oil of relative density 0.96.

The fuel oil is taken into Nos. 2, 3 and 5 double bottom tanks, filling Nos. 3 and 5, and leaving No. 2 slack.

The ship then sails on a 20-day passage consuming 30 tonnes of fuel oil per day. On arrival at her destination Nos. 2 and 3 tanks are empty, and the remaining fuel oil is in No. 5 tank. Find the ship's GM's for the departure and arrival conditions.

Dimensions of the tanks:

No. 2 15×15 m × 1 m
No. 3 22 m × 15 m × 1 m
No. 5 12 m × 15 m × 1 m

Assume that the KM is constant and that the KG of the fuel oil in every case is half of the depth of the tank.

6 A ship's displacement is 5100 tonnes, KG = 4 m, and KM = 4.8 m. A double-bottom tank on the starboard side is 20 m long, 6 m wide and 1 m deep and is full of fresh water. Calculate the list after 60 tonnes of this water has been consumed.

7 A ship of 6000 tonnes displacement has KG 4 m and KM 4.5 m. A double-bottom tank in the ship 20 m long and 10 m wide is partly full of salt-water

ballast. Find the moment of statical stability when the ship is heeled 5 degrees.

8 A box-shaped vessel has the following data.

Length is 80 m, breadth is 12 m, draft even keel is 6 m, KG is 4.62 m.

A double bottom tank 10 m long, of full width and 2.4 m depth is then half-filled with water ballast having a density of 1.025 t/m^3. The tank is located at amidships.

Calculate the new even keel draft and the new transverse GM after this water ballast has been put in the double bottom tank.

Chapter 21
Bilging and permeability

Bilging amidships compartments

When a vessel floats in still water it displaces its own weight of water. Figure 21.1(a) shows a box-shaped vessel floating at the waterline WL. The weight of the vessel (W) is considered to act downwards through G, the centre of gravity. The force of buoyancy is also equal to W and acts upwards through B, the centre of buoyancy. b = W.

Now let an empty compartment amidships be holed below the waterline to such an extent that the water may flow freely into and out of the compartment. A vessel holed in this way is said to be 'bilged'.

Figure 21.1(b) shows the vessel in the bilged condition. The buoyancy provided by the bilged compartment is lost. The draft has increased and the vessel now floats at the waterline W_1L_1, where it is again displacing its own

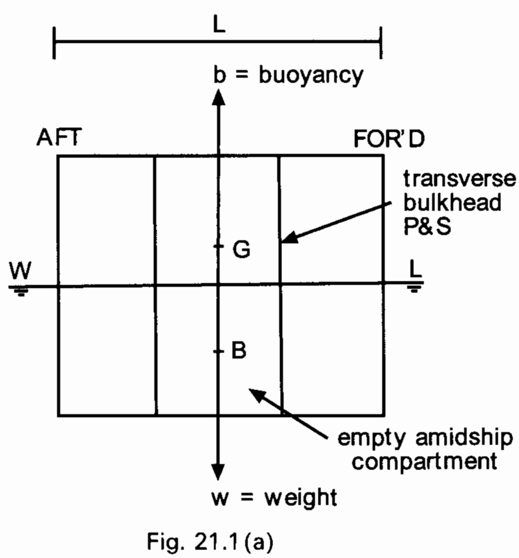


Fig. 21.1 (a)

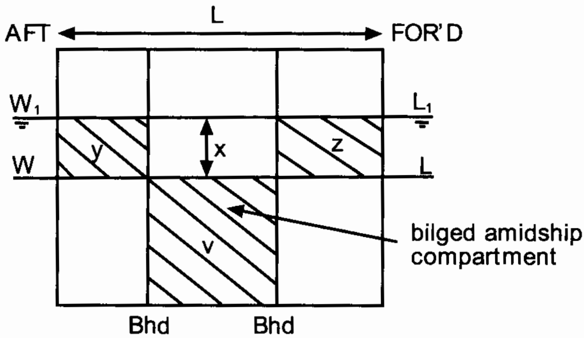


Fig. 21.1 (b)

weight of water. 'X' represents the increase in draft due to bilging. The volume of lost buoyancy (v) is made good by the volumes 'y' and 'z'.

$$\therefore\ v = y + z$$

Let 'A' be the area of the water-plane before bilging, and let 'a' be the area of the bilged compartment. Then:

$$y + z = Ax - ax$$

or

$$v = x(A - a)$$

$$\text{Increase in draft} = x = \frac{v}{A - a}$$

i.e.

$$\text{Increase in draft} = \frac{\text{Volume of lost buoyancy}}{\text{Area of intact waterplane}}$$

Note. Since the distribution of weight within the vessel has not been altered the KG after bilging will be the same as the KG before bilging.

Example 1

A box-shaped vessel is 50 metres long and is floating on an even keel at 4 metres draft. A compartment amidships is 10 metres long and is empty. Find the increase in draft if this compartment is bilged. See Fig. 21.1(c).

$$x = \frac{v}{A - a} = \frac{l \times B \times B}{(L - l)B}$$

let

$$B = \text{Breadth of the vessel}$$

then

$$x = \frac{10 \times B \times 4}{50 \times B - 10 \times B}$$

$$= \frac{40B}{40B}$$

Increase in draft = 1 metre

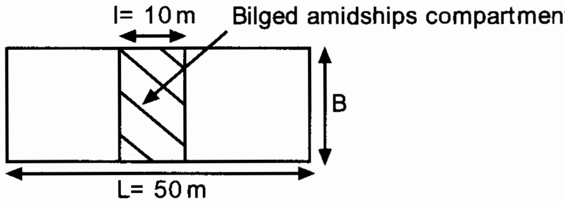


Fig. 21.1 (c)

Example 2

A box-shaped vessel is 150 metres long × 24 metres wide × 12 metres deep and is floating on an even keel at 5 metres draft. GM = 0.9 metres. A compartment amidships is 20 metres long and is empty. Find the new GM if this compartment is bilged.

$$\text{Old KB} = \tfrac{1}{2}\ \text{Old draft}$$
$$= 2.5\,\text{m}$$
$$\text{Old BM} = B^2/12d$$
$$= \frac{24 \times 24}{12 \times 5}$$
$$= 9.6\,\text{m}$$
$$\text{Old KB} = +2.5\,\text{m}$$
$$\text{Old KM} = 12.1\,\text{m}$$
$$\text{Old GM} = -0.9\,\text{m}$$
$$\text{KG} = 11.2\,\text{m}$$

This KG will not change after bilging has taken place.

$$x = \frac{v}{A - a}$$
$$= \frac{20 \times 24 \times 5}{150 \times 24 - 20 \times 24}$$
$$= \frac{2400}{130 \times 24}$$
$$\text{Increase in draft} = 0.77\,\text{m}$$
$$\text{Old draft} = 5.00\,\text{m}$$
$$\text{New draft} = \underline{5.77\,\text{m}} = \text{say draft } d_2$$

$$\text{New KB} = \frac{1}{2}\text{ New draft} = \frac{d_2}{2}$$
$$= 2.89\text{ m}$$
$$\text{New BM} = B^2/12d_2$$
$$= \frac{24 \times 24}{12 \times 5.77}$$
$$= 8.32\text{ m}$$
$$\text{New KB} = +\ 2.89\text{ m}$$
$$\text{New KM} = 11.21\text{ m}$$
$$\text{As before, KG} = -\ 11.20\text{ m}$$

Ans. New GM = 0.01 m
This is +ve but dangerously low in value!!

Permeability μ

Permeability is the amount of water that can enter a compartment or tank after it has been bilged. When an empty compartment is bilged, the whole of the bouyancy provided by that compartment is lost. Typical values for permeability μ are as follows:

Empty compartment	μ = 100%
Engine room	μ = 80% to 85%
Grain filled cargo hold	μ = 60% to 65%
Coal filled compartment	μ = 36% approx
Filled water ballast tank (when ship is in salt water)	μ = 0%

Consequently, the higher the value of the permeability for a bilged compartment, the greater will be a ship's loss of bouyancy when the ship is bilged.

The permeability of a compartment can be found from the formula:

$$\mu = \text{Permeability} = \frac{\text{Broken Stowage}}{\text{Stowage Factor}} \times 100 \text{ per cent}$$

The broken stowage to be used in this formula is the broken stowage per tonne of stow.

When a bilged compartment contains cargo, the formula for finding the increase in draft must be amended to allow for the permeability. If 'μ' represents the permeability, expressed as a fraction, then the volume of lost buoyancy will be 'μv' and the area of the intact waterplane will be 'A − μv' square metres. The formula then reads:

$$x = \frac{\mu v}{A - \mu a}$$

Example 3

A box-shaped vessel is 64 metres long and is floating on an even keel at 3 metres draft. A compartment amidships is 12 m long and contains cargo having a permeability of 25 per cent. Calculate the increase in the draft if this compartment be bilged.

$$x = \frac{\mu v}{A - \mu a}$$

$$= \frac{\frac{1}{4} \times 12 \times B \times 3}{64 \times B - \frac{1}{4} \times 12 \times B}$$

$$= \frac{9B}{61B}$$

Ans. Increase in draft = 0.15 m

Example 4

A box-shaped vessel 150 m × 20 m × 12 m is floating on an even keel at 5 metres draft. A compartment amidships is 15 metres long and contains timber of relative density 0.8, and stowage factor 1.5 cubic metres per tonne. Calculate the new draft if this compartment is now bilged.

The permeability 'μ' must first be found by using the formula given above. i.e.

$$\text{Permeability} = \frac{BS}{SF} \times 100 \text{ per cent} = \text{'}\mu\text{'}$$

The stowage factor is given in the question. The broken stowage per tonne of stow is now found by subtracting the space which would be occupied by one tonne of solid timber from that actually occupied by one tonne of timber in the hold. One tonne of fresh water occupies one cubic metre and the relative density of the timber is 0.8.

$$\therefore \text{Space occupied by one tonne of solid timber} = \frac{1}{0.8}$$

$$= 1.25 \text{ cubic metres}$$

$$\text{Stowage Factor} = 1.50 \text{ cubic metres}$$

$$\therefore \text{Broken Stowage} = 0.25 \text{ cubic metres}$$

$$\text{Permeability '}\mu\text{'} = \frac{BS}{SF} \times 100 \text{ per cent}$$

$$= \frac{0.25}{1.50} \times 100 \text{ per cent}$$

$$= 100/6 \text{ per cent}$$

$$\therefore \text{'}\mu\text{'} = 1/6 \text{ or } 16.67 \text{ per cent}$$

$$\text{Increase in draft} = x = \frac{\mu v}{A - \mu a}$$

$$= \frac{1/6 \times 15 \times 20 \times 5}{150 \times 20 - 1/6 \times 15 \times 20}$$

$$= 250/2950 = 0.085 \text{ m}$$

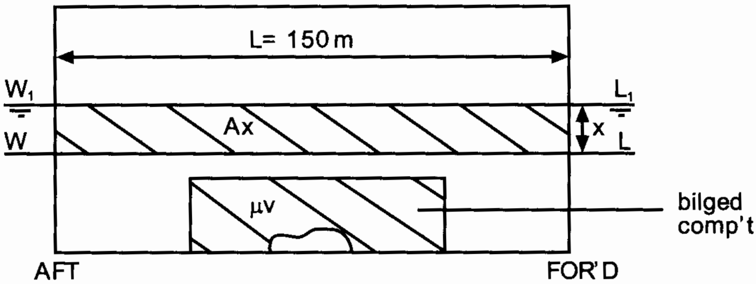


Fig. 21.2

Increase in draft = 0.085 metres

Old draft = 5.000 metres = draft d_1

Ans. New draft = 5.085 metres = draft d_2

When the bilged compartment does not extend above the waterline, the area of the intact waterplane remains constant as shown in Figure 21.2.

In this figure:

$$\mu v = Ax$$

Let

d = Density of the water, then

$$\mu v \times d = Ax \times d$$

but

$\mu v \times d$ = Mass of water entering the bilged compartment, and

$Ax \times d$ = Mass of the extra layer of water displaced.

Therefore, when the compartment is bilged, the extra mass of water displaced is equal to the buoyancy lost in the bilged compartment. It should be carefully noted, however, that although the effect on draft is similar to that of loading a mass in the bilged compartment equal to the lost buoyancy, no mass has in fact been loaded. The *displacement* after bilging is *the same* as the displacement before bilging and there is *no alteration* in the position of the vessel's *centre of gravity*. The increase in the draft is due solely to lost buoyancy.

Example 5

A ship is floating in salt water on an even keel at 6 metres draft. TPC is 20 tonnes. A rectangular-shaped compartment amidships is 20 metres long, 10 metres wide, and 4 metres deep. The compartment contains cargo with permeability 25 per cent. Find the new draft if this compartment is bilged.

$$\text{Buoyancy lost} = \frac{25}{100} \times 20 \times 10 \times 4 \times 1.025 \text{ tonnes}$$

$$= 205 \text{ tonnes}$$

Extra mass of water displaced = TPC × X tonnes

$$\therefore X = w/TPC$$
$$= 205/20$$
$$\text{Increase in draft} = 10.25 \text{ cm}$$
$$= 0.1025 \text{ m}$$
$$\text{plus the old draft} = \underline{6.0000 \text{ m}}$$

Ans. New draft = 6.1025 m

Note: The *lower* the permeability is the *less* will be the changes in end drafts after bilging has taken place.

Bilging end compartments

When the bilged compartment is situated in a position away from amidships, the vessel's mean draft will increase to make good the lost buoyancy but the trim will also change.

Consider the box-shaped vessel shown in Figure 21.3(a). The vessel is floating upright on an even keel, WL representing the waterline. The centre of buoyancy (B) is at the centre of the displaced water and the vessel's centre of gravity (G) is vertically above B. There is no trimming moment.

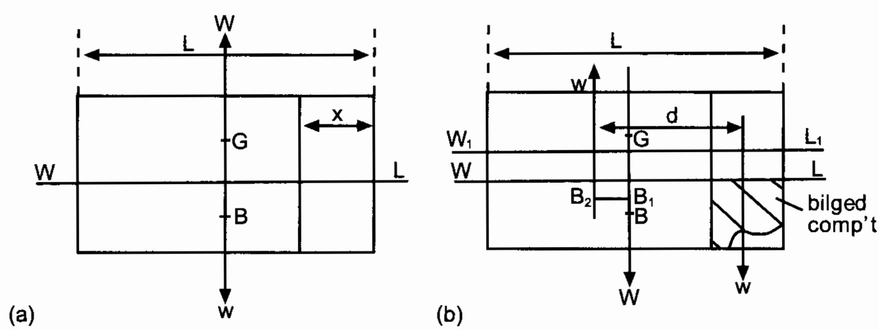


Fig. 21.3

Now let the forward compartment which is X metres long be bilged. To make good the loss in buoyancy, the vessel's mean draft will increase as shown in Figure 21.3(b), where W_1L_1 represents the new waterline. Since there has been no change in the distribution of mass within the vessel, the centre of gravity will remain at G. It has already been shown that the effect on mean draft will be similar to that of loading a mass in the compartment equal to the mass of water entering the bilged space to the original waterline.

The vertical component of the shift of the centre of buoyancy (B to B_1) is due to the increase in the mean draft. KB_1 is equal to half of the new draft. The horizontal component of the shift of the centre of buoyancy (B_1B_2) is equal to X/2.

A trimming moment of $W \times B_1B_2$ by the head is produced and the vessel will trim about the centre of flotation (F), which is the centre of gravity of the new water-plane area.

$$B_1B_2 = \frac{w \times d}{W}$$

or

$$W \times B_1B_2 = w \times d$$

but

$$W \times B_1B_2 = \text{Trimming moment},$$

$$\therefore\ w \times d = \text{Trimming moment}$$

It can therefore be seen that the effect on trim is similar to that which would be produced if a mass equal to the lost buoyancy were loaded in the bilged compartment.

Note. When calculating the TPC, MCTC, etc., it must be remembered that the information is required for the vessel in the bilged condition, using draft d_2 and intact length l_2.

Example 6

A box-shaped vessel 75 metres long × 10 metres wide × 6 metres deep is floating in salt water on an even keel at a draft of 4.5 metres. Find the new drafts if a forward compartment 5 metres long is bilged.

(a) First let the vessel sink bodily.

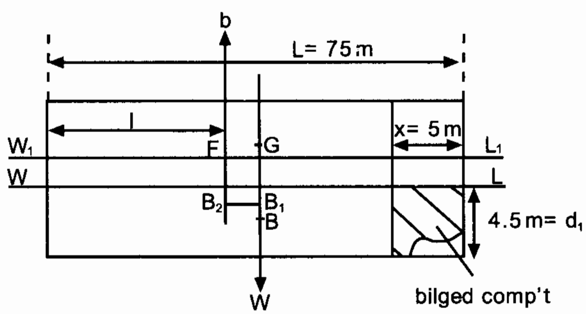


Fig. 21.4

$$w = x \times B \times d_1 \times 1.025 \text{ tonnes}$$
$$= 5 \times 10 \times 4.5 \times 1.025$$
$$w = 230.63 \text{ tonnes}$$

$$TPC = \frac{WPA}{97.56} = \frac{L_2 \times B}{97.56}$$
$$= \frac{70 \times 10}{97.56}$$
$$TPC = 7.175$$

$$\text{Increase in draft} = w/TPC$$
$$= 230.63/7.175$$
$$= 32.14\,\text{cm}$$
$$= 0.321\,\text{m}$$
$$+$$
$$\text{Old draft} = 4.500\,\text{m} = \text{draft } d_1$$
$$\text{New mean draft} = 4.821\,\text{m} = \text{draft } d_2$$

(b) Now consider the trim.

$$W = L \times B \times d_1 \times 1.025 \text{ tonnes}$$
$$= 75 \times 10 \times 4.5 \times 1.025$$
$$W = 3459 \text{ tonnes}$$

$$BM_L = I_L/V$$
$$= \frac{BL_2^3}{12V}$$
$$= \frac{10 \times 70^3}{12 \times 75 \times 10 \times 4.5}$$
$$BM_L = 84.7 \text{ metres}$$

$$MCTC \simeq \frac{W \times BM_L}{100L}$$
$$= \frac{3459 \times 84.7}{100 \times 75}$$
$$= 39.05 \text{ tonnes m per cm}$$

$$\text{Change of trim} = \frac{\text{Moment changing trim}}{MCTC}$$

where

$$d = \frac{LBP}{2} = \frac{75}{2} = 37.5\,\text{m} = \text{lever from new LCF}$$
$$= \frac{230.6 \times 37.5}{39.05}$$
$$= 221.4\,\text{cm by the head}$$

After bilging, LCF has moved to F, i.e. $(L - x)/2$ from the stern

$$\text{Change of draft aft} = \frac{l}{L} \times \text{Change of trim}$$
$$= \frac{35}{75} \times 221.4$$
$$= 103.3\,\text{cm} = 1.033\,\text{m}$$

$$\text{Change of draft forward} = \frac{40}{75} \times 221.4$$
$$= 118.1\,\text{cm} = 1.181\,\text{m}$$

(c) Now find new drafts.

Drafts before trimming	A 4.821 m	F 4.821 m
Change due to trim	−1.033 m	+1.181 m
Ans. New Drafts	A 3.788 m	F 6.002 m

Example 7

A box-shaped vessel 100 metres long × 20 metres wide × 12 metres deep is floating in salt water on an even keel at 6 metres draft. A forward compartment is 10 metres long, 12 metres wide and extends from the outer bottom to a watertight flat, 4 metres above the keel. The compartment contains cargo of permeability 25 per cent. Find the new drafts if this compartment is bilged.

Mass of water entering the bilged compartment $= \frac{25}{100} \times 10 \times 12 \times 4 \times 1.025$
$= 123$ tonnes

$$TPC_{SW} = \frac{WPA}{97.56} = \frac{100 \times 20}{97.56}$$

TPC = 20.5 tonnes

$$\text{Increase in draft} = w/TPC = 123/20.5 = 6\text{ cm}$$

Increase in draft = 0.06 m

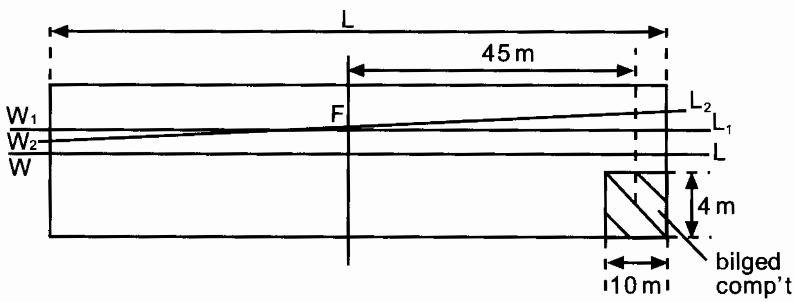


Fig. 21.5

$$W = L \times B \times d_1 \times 1.025$$
$$= 100 \times 20 \times 6 \times 1.025$$
$$W = 12\,300 \text{ tonnes}$$

$$BM_L = \frac{I_L}{V} = \frac{BL^3}{12V} = \frac{B \times L^3}{12 \times L \times B \times d} = \frac{L^2}{12 \times d_1}$$

$$BM_L = \frac{100 \times 100}{12 \times 6}$$

$BM_L = 139$ metres

$$MCTC \simeq \frac{W \times BM_L}{100 \times L} = \frac{12\,300 \times 139}{100 \times 100}$$

$$\text{MCTC} = 171 \text{ tonnes m per cm}$$

$$\text{Trimming moment} = W \times B_1B_2$$
$$= w \times d$$

$$\text{Trimming moment} = 123 \times 45 \text{ tonnes m}$$

$$\text{Change of trim} = \frac{\text{Trimming moment}}{\text{MCTC}} = \frac{123 \times 45}{171}$$

$$\text{Change of trim} = 32.4 \text{ cm by the head,}$$

i.e. 0.32 m by the head

Note. The centre of flotation, being the centroid of the water-plane area, remains amidships.

Old drafts	A	6.00 m	F	6.00 m
Bodily increase		+0.06 m		+0.06 m
		6.06 m		6.06 m
Change due to trim		−0.16 m		+0.16 m
Ans. New Drafts	A	5.90 m	F	6.22 m

Effect of bilging on stability

It has already been shown that when a compartment in a ship is bilged the mean draft is increased. The change in mean draft causes a change in the positions of the centre of buoyancy and the initial metacentre. Hence KM is changed and, since KG is constant, the GM will be changed.

Example 8

A box-shaped vessel 40 metres long, 8 metres wide and 6 metres deep, floats in salt water on an even keel at 3 metres draft. GM = 1 metre. Find the new GM if an empty compartment 4 metres long and situated amidships is bilged.

(a) Original condition before bilging.

Find the KG

$$KB = \frac{d_1}{2} = 1.5 \text{ metres}$$

$$BM = \frac{I}{V} = \frac{LB^3}{12V} = \frac{B^2}{12 \times d_1} = \frac{8 \times 8}{12 \times 3}$$

BM =	1.78 m
KB =	+1.50 m
KM =	3.28 m
GM =	−1.00 m
KG =	2.28 m

(b) Vessel's condition after bilging.

Find the New Draft

$$\text{Lost buoyancy} = 4 \times 8 \times 3 \times 1.025 \text{ tonnes}$$

$$TPC_{SW} = \frac{WPA}{97.56} = \frac{36 \times 8}{97.56}$$

$$\text{Increase in draft} = \frac{\text{Lost buoyancy}}{TPC}$$

$$= 4 \times 8 \times 3 \times 1.025 \times \frac{100}{36 \times 8 \times 1.025} \text{ cm}$$

$$= 33.3 \text{ cm or } 0.33 \text{ m}$$

It should be noted that the increase in draft can also be found as follows:

$$\text{Increase in draft} = \frac{\text{Volume of lost buoyancy}}{\text{Area of intact water-plane}} = \frac{4 \times 8 \times 3}{36 \times 8}$$

$$= 1/3 \text{ metres.}$$

$$\text{Original draft} = 3.000 \text{ m} = \text{draft } d_1$$

$$\text{New draft} = 3.333 \text{ m} = \text{draft } d_2$$

(c) Find the New GM

$$KB = \frac{d_2}{2} = 1.67 \text{ m}$$

$BM = I/V$ (Note: 'I' represents the second moment of the intact water-plane about the centre line)

$$= \frac{(L - l)B^3}{12 \times V} = \frac{36 \times 8^3}{12 \times 40 \times 8 \times 3}$$

$$\begin{aligned} BM_2 &= 1.60 \text{ m} \\ &+ \\ KB_2 &= 1.67 \text{ m} \\ KM_2 &= 3.27 \text{ m} \\ &- \\ KG &= 2.28 \text{ m as before bilging occurred} \\ \text{Final } GM_2 &= 0.99 \text{ m} \end{aligned}$$

GM_2 is +ve so vessel is in stable equilibrium.

Summary

When solving problems involving bilging and permeability it is suggested that:

1 Make a sketch from given information.
2 Calculate mean bodily sinkage using w and TPC.
3 Calculate change of trim using GM_L or BM_L.
4 Collect calculated data to evaluate the final requested end drafts.

Exercise 21

Bilging amidships compartments

1 (a) Define permeability, 'μ'.
(b) A box-shaped vessel 100 m long, 15 m beam floating in salt water, at a mean draft of 5 m, has an amidships compartment 10 m long which is loaded with a general cargo. Find the new mean draft if this compartment is bilged, assuming the permeability to be 25 per cent.

2 A box-shaped vessel 30 m long, 6 m beam, 5 m deep, has a mean draft of 2.5 m. An amidships compartment 8 m long is filled with coal stowing at 1.2 cu. m per tonne. 1 cu. m of solid coal weighs 1.2 tonnes. Find the increase in the draft if the compartment is holed below the waterline.

3 A box-shaped vessel 60 m long, 15 m beam, floats on an even keel at 3 m draft. An amidships compartment is 12 m long and contains coal (SF = 1.2 cu. m per tonne and relative density = 1.28). Find the increase in the draft if this compartment is bilged.

4 A box-shaped vessel 40 m long, 6 m beam, is floating at a draft of 2 m F and A. She has an amidships compartment 10 m long which is empty. If the original GM was 0.6 m, find the new GM if this compartment is bilged.

5 If the vessel in Question 4 had cargo stowed in the central compartment such that the permeability was 20 per cent, find the new increase in the draft when the vessel is bilged.

6 A box-shaped vessel 60 m × 10 m × 6 m floats on an even keel at a draft of 5 m F and A. An amidships compartment 12 m long contains timber of relative density 0.8 and stowage factor 1.4 cu. m per tonne. Find the increase in the draft if this compartment is holed below the waterline.

7 A box-shaped vessel 80 m × 10 m × 6 m is floating upright in salt water on an even keel at 4 m draft. She has an amidships compartment 15 m long which is filled with timber (SF = 1.5 cu. m per tonne). 1 tonne of solid timber would occupy 1.25 cu. m of space. What would be the increase in the draft if this compartment is now bilged?

Bilging end compartments

8 A box-shaped vessel 75 m × 12 m is floating upright in salt water on an even keel at 2.5 m draft F and A. The forepeak tank which is 6 m long is empty. Find the final drafts if the vessel is now holed forward of the collision bulkhead.

9 A box-shaped vessel 150 m long, 20 m beam, is floating upright in salt water at drafts of 6 m F and A. The collision bulkhead is situated 8 m from forward. Find the new drafts if the vessel is now bilged forward of the collision bulkhead.

10 A box-shaped vessel 60 m long, 10 m beam, is floating upright in salt water on even keel at 4 m draft F and A. The collision bulkhead is 6 m

from forward. Find the new drafts if she is now bilged forward of the collision bulkhead.

11 A box-shaped vessel 65 m × 10 m × 6 m is floating on an even keel in salt water at 4 m draft F and A. She has a forepeak compartment 5 m long which is empty. Find the new drafts if this compartment is now bilged.

12 A box-shaped vessel 64 m × 10 m × 6 m floats in salt water on an even keel at 5 m draft. A forward compartment 6 metres long and 10 metres wide, extends from the outer bottom to a height of 3.5 m, and is full of cargo of permeability 25 per cent. Find the new drafts if this compartment is now bilged.